A STUDY OF A HYDRO MAGNETIC FLUID FLOW IN A ROTATING SYSTEM BOUNDED BY TWO PARALLEL PLATES

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A study of a hydro magnetic fluid flow in a rotating system bounded by two parallel plates

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DECLARATION

This thesis is my original work and has not been presented for a degree in any other University.

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DEDICATION

To my father Mr. Ezra Kirima and my mother Mrs. Alice Kirima for their parental guidance and their support towards my education. To my wife Dorcas for her support and to my son James for his patience.

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NOMENCLATURE

ROMAN SYMBOLS

$\boldsymbol{B}$	Magnetic Induction, Wbm^{-2}
B_0	Fixed magnetic induction, Wbm^{-2}
C	Dimensionless concentration of injected material
C_1^*, C_2^*	Species concentration at the plate,kgm^{-3}
C_p	Specific heat at constant pressure, $Jkg^{-1}K^{-1}$
D	Diffusion coefficient, m^2s^{-1}
$\frac{D}{Dt}$	Material or substantive operator $\left(= \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)$
$\mathbf{E}$	Electric field, Vm^{-1}
E_x, E_y, E_z	Components of electric field, Vm^{-1}
Er	Rotational parameter$\left(= \frac{\Omega v}{U_0^2}\right)$
Ec	Eckert number$\left(= \frac{U_0^2}{C_P(T_2^* - T_1^*)}\right)$
e	Electron number density
F	Body force, N
Fe	Electromagnetic force, N
Fg	Non electric force per unit volume, N
g	Gravitational acceleration, ms^{-2}

Gr	Grashof number $\left(= \frac{\upsilon g\beta(T_2^* - T_1^*)}{U_0^3}\right)$
Gc	Modified Grashof number $\left(= \frac{\upsilon g\beta_c(C_2^* - C_1^*)}{U_0^3}\right)$
h	Specific enthalpy $KJkg^{-1}$
$\mathbf{H}$	Magnetic field intensity, Wbm^{-2}
H_x, H_y, H_z	Components of magnetic field intensity, Wbm^{-2}
H_0	Fixed magnetic field intensity, Wbm^{-2}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the x^*, y^* and z^* directions respectively
i	Complex number $(= \sqrt{-1})$
J	Electric current density, Am^{-2}
j_x, j_y, j_z	Components of electric current density, Am^{-2}
$\mathbf{j}_i$	Flow rate per unit area (diffusion flux), $Kgs^{-1}m^{-2}$
K	Thermal conductivity $Wm^{-1}K^{-1}$
L	Characteristic length, m
M	Hartmann number $\left(= \frac{\sqrt{\sigma\upsilon\mu_e^2 H_0^2}}{\sqrt{\rho U_0^2}}\right)$
M_1	Magnetic parameter $\left(= \frac{\sqrt{\sigma\mu_e^2 H_0^2 \upsilon}}{\sqrt{\mu U_0^2/\upsilon}}\right)$
Nu	Nusselt number $\left(= \frac{\partial\theta}{\partial z}\Big|_{z=\pm L}\right)$
P	Pressure of the fluid, Nm^{-2}
P_0	Representative value of pressure, Nm^{-2}

Pr	Prandtl number$\left(=\frac{\mu C_p}{K}\right)$
$\mathbf{Q}$	Heat flux vector, Wm^{-2}
$\mathbf{q}$	Velocity vector of the fluid,ms^{-1}
Re	Reynold's number $\left(=\frac{\rho U_0 L}{\mu}\right)$
Rm	Magnetic Reynold's number $\left(=\frac{U_0 L}{\upsilon_H}\right)$
R_p	Pressure parameter$\left(=\frac{P_0}{\rho U_0^2}\right)$
S	Specific entropy, $KJKg^{-1}$
Sc	Schmidt number$\left(=\frac{\upsilon}{D}\right)$
Sh	Sherwood number $\left(=\frac{\partial C}{\partial z}\Big\|_{z=\pm L}\right)$
T^*	Dimensional Temperature of fluid, K
t^*	Dimensional time, s
t	Dimensionless time
t_0	Representative time, s
T_1^*, T_2^*	Temperature of fluid at the plates, K
u^*, v^*, w^*	Dimensional velocity components, ms^{-1}
U_0	Velocity of moving plate, ms^{-1}
V	Fluid volume, m^3
w_0^*	Dimensional suction velocity, ms^{-1}
w_0	Dimensionless suction velocity
x^*, y^*, z^*	Cartesian co-ordinates in dimensional form, m

GREEK SYMBOLS

β Coefficient of volumetric expansion, K^{-1}

β_c Coefficient of expansion due to concentration gradients,K^{-1}

θ Dimensionless fluid temperature

ρ Fluid density, kgm^{-3}

μ Coefficient of viscosity, $kgm^{-1}s^{-1}$

ν Kinematic Viscosity, m^2s^{-1}

μ_e Magnetic permeability, Hm^{-1}

σ Electrical conductivity, $\Omega^{-1}m^{-1}$

Ω Angular velocity, s^{-1}

ρ_e Excess electrical charge

$\delta_{i,j}$ Kronecker delta $\left(= \begin{cases} 1, i = j \\ 0, otherwise \end{cases}\right)$

ρ_∞ Free stream fluid density, kgm^{-3}

Δt Time interval, s

Δz Distance interval, m

$\varepsilon_{i,j}$ Rate of strain tensor

κ Thermal diffusivity $\left(= \frac{K}{\rho c_p}\right)$

$\sigma_{i,j}$ Stress tensor, Nm^{-2}

∇ Gradient operator$\left(= \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right)$

η Cyclotron frequency, Hz

ω_e Electron cyclotron frequency, Hz

τ_e Collision time of electrons, s

η_e Number density of electrons

ϕ Viscous dissipation function, s^{-2}

ABSTRACT

An electrically conducting fluid flowing between two parallel plates has been considered in this study. The fluid flow is unsteady and a magnetic field is applied perpendicular to the plates. Various configurations have been considered for the plates with either one plate moving or both plates moving in opposite directions relative to each other. The whole system is then rotated in a direction normal to the plates with a constant angular velocity Ω. Temperature and concentration gradients have been considered across the plates. The Hall current effect has also been put into consideration when studying the fluid flow. The effects of changing various parameters on the velocity, temperature and concentration profiles has been discussed. These parameters include the rotation parameter Er, the Schmidt number Sc, the Hall parameter m, the suction parameter S, the Eckert number Ec, the pressure gradient $\frac{dP}{dx}$ and time t. The results that are obtained are then presented on graphs and the observations are discussed. Later the effect of changing these parameters on the skin friction, the rate of heat transfer and the rate of mass transfer are studied. The results that are obtained are presented in tables and then discussed.

The effect of changing the parameters mentioned above is observed either to increase, to decrease or to have no effect on the velocity profiles, the temperature profiles, the concentration profiles, the skin friction and the rates of heat and mass transfer.

Chapter 1

INTRODUCTION

Fluid dynamics is an area of study that deals with the flow of fluids. The fluids in consideration include gases, liquids and ionized gases also called plasma. Electromagnetism on the other hand studies the interaction between the electric and magnetic fields. Magnetofluiddynamics (MFD) is an area of study that combines the two areas of fluid dynamics and electromagnetism. Magnetohydrodynamics (MHD) is a narrower area of study than MFD because MHD combines the two areas of study of flow of liquids called hydrodynamics and study of electromagnetism. In that case, MHD excludes the area of study of gases in motion called gas dynamics and also the study of plasmas.

1.1 Definition of terms

1.1.1 Newtonian and non-newtonian fluids

A fluid is the state of matter that cannot resist a small shear force or stress without being deformed. A fluid can be a liquid, a gas, or ionized gases called plasma. Further, if the stresses associated with fluid motion depend linearly on the instantaneous value of the rate of deformation, the fluid in consideration is termed as a newtonian fluid. Examples of newtonian fluids include water, sugar solutions, glycerine, light hydrocarbon oils, air and other gases. The fluids whose viscosity changes with the applied shear force are called non-newtonian fluids. Consequently these non newtonian fluids may not have a well defined viscosity. Examples of non-newtonian fluids include mud, milk, blood, bitumen, concentrated solution of sugar and water, suspensions of rice starch and corn starch and non drip paints.

1.1.2 Forces acting on the fluid

The forces acting on a fluid element are classified as either body forces which are forces acting at a distance on a fluid particle or surface forces which are forces due to direct contact of a particle with other fluid particles or solid walls. Some of the body forces considered in MFD include the electric force and the magnetic force in addition to other forces like viscous, pressure, gravity and inertia forces studied in fluid dynamics. This is defined in section 1.1 on the preceding page.

1.1.3 Heat transfer

Heat transfer involves the study of energy transfer taking place between material bodies as a result of temperature difference. The different modes of heat transfer include conduction, convection and radiation.

(a) *Conduction heat transfer:* This is the energy transfer from the more energetic to the less energetic particles of a substance as a result of interactions between the particles.

(b) *Radiation heat transfer:* This is the energy emitted by matter due to changes in electron configuration of the constituent atoms or molecules. It is transported by electromagnetic waves (or alternatively photons).

(c) *Convection heat transfer:* If the heat transport process is affected by the flow of a fluid such that two different portions of a fluid mix then the mode of heat transfer is termed as convection. This mode of heat transfer can further be classified as free, forced or mixed convection. In free or natural convection, fluid motion is as a result of density gradients created by temperature or concentration gradients existing in the fluid mass. Forced convection fluid motion takes place due to external forces such as those from a pump or fan acting on the fluid. A special case called mixed convection arises when both free and forced convection fluid motions exist simultaneously.

1.1.4 Viscous fluid and skin friction

Viscosity is a measure of internal friction of a fluid or the resistance to deformation. This occurs due to molecules from a region of high bulk velocity colliding with molecules moving with lower bulk velocity and vice versa. Skin friction is a drag force due to viscous resistance tangential to the surface on which the fluid is flowing.

1.1.5 Boundary layers

Heat transfer can occur between a fluid in motion and a bounding surface when the two are at different temperatures. This interaction leads to the development of thin fluid layers adjacent to the surface of a body or a solid wall in which strong viscous effects exist. These layers are called boundary layers. Outside the boundary layer is a free stream, which is a flow region that is not affected by the obstruction, heating effect or mass transport introduced at the solid wall. The laminar boundary layer is a flow region where molecules move from one lamina to another carrying with them a momentum corresponding to the velocity of the flow. Depending on the nature of the fluid flow in consideration, we can study either one or a combination of the boundary layers discussed below.

(a) *Velocity boundary layer:* Fluid particles in contact with a stationary surface assume zero tangential velocity. Similarly, fluid particles in contact with a moving surface will move at the velocity of the surface. In fluid dynamics this phenomenon is called the no slip condition. When a fluid flows, there occurs a net momentum transport from regions of high velocity to regions of low velocity thus creating a viscous shear stress in the direction of the flow. The significance of the velocity boundary layer is to determine the surface (or skin) friction of the fluid.

(b) *Thermal boundary layer*: The thermal boundary layer develops due to the presence of temperature gradients between the surface and the free stream region. The thermal boundary layer is important in determining the rate of convection heat transfer.

(c) *Concentration boundary layer:* A concentration boundary layer develops in a fluid region where concentration gradients exist between the surface and the free stream. The significance of this boundary layer is in determining the rate of convection mass transfer.

1.1.6 Mass transfer

In mass transfer by convection, fluid motion on the larger scale combines with mass diffusion on a molecular scale to promote the transport of a species for which a concentration gradient exists. When the solid boundary allows fluid to pass through it, then we are considering a porous medium. A porous medium is said to be homogenous if the ratio of the pore area to the total area of the solid boundary is a constant. Otherwise the medium is termed as non homogeneous.

1.1.7 Steady and unsteady fluid flow

When the flow variables such as density, velocity and pressure as well as the thermodynamic properties vary with respect to time, the flow in consideration is termed as unsteady flow. Otherwise if these variables are independent of time then the fluid flow is referred to as a steady fluid flow.

1.1.8 Compressible and incompressible fluids

A fluid is said to be compressible if its density changes appreciably (typically by a few percent) within the domain of interest. Typically, this will occur when the fluid velocity exceeds Mach 0.3. Hence, low velocity fluid flows behave as incompressible. An incompressible fluid is one whose density is constant everywhere. In many practical cases of fluid flow, the variation of density of the fluid may be neglected. This is particularly important when the fluid flow is due to buoyant forces and in such cases, the fluid is considered to be incompressible.

1.1.9 Laminar and turbulent fluid flow

Laminar fluid flow is an organized flow field that can be described with streamlines where the fluid flows in layers which do not mix. In order for laminar flow to be permissible, the viscous stresses must dominate over the fluid inertia stresses. A flow field that cannot be described with streamlines in the absolute sense is termed as turbulent. However, time-averaged streamlines can be defined to describe the average behavior of the flow. In turbulent flow, the inertia stresses dominate over the viscous stresses, leading to small-scale chaotic behavior in the fluid motion.

1.1.10 Rotational and irrotational fluid flow

An irrotational fluid flow is one whose streamlines never loop back on themselves. Typically, only inviscid fluids can be irrotational. A rotational fluid flow can contain streamlines that loop back on themselves. Hence, fluid particles following such streamlines will travel along closed paths. Bounded (and hence nonuniform) viscous fluids exhibit rotational flow, typically within their boundary layers. Since all real fluids are viscous to some extent, all real fluids exhibit a level of rotational flow somewhere in their domain. Regions of rotational flow correspond to the regions of viscous losses in a fluid.

1.1.11 Magnetofluiddynamics (MFD)

Faraday's law of electromagnetic induction reveals that when a conductor is moved across a magnetic field, an electromotive force is produced in the conductor. Also, when a current carrying conductor is moved in a magnetic field it experiences a force that tends to move it at right angles to the electric field. Lenz's law gives the direction of the induced current as that direction that opposes the change in the magnetic field that causes it. When an electrically conducting fluid moves past a magnetic field, there arises an interaction between the flow field and the magnetic field. The magnetic field exerts a force on the fluid due to induced currents and the induced currents affect the original magnetic field. If these electromagnetic forces generated in this way are of the same order of magnitude as the hydrodynamic forces then these electromagnetic forces have to be taken into account when considering the flow field. MFD fluid flows involve the flow of electrically conducting fluids in the presence of a magnetic field. MFD therefore is a field of study involving both electromagnetic theory and fluid dynamics.

1.2 Review of related literature

The study of MHD fluid flow between parallel plates has been carried out for a long period of time. Hartmann & Lazarus (1937) studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated plates. Since then, many researchers have been involved in the study of fluid flows between parallel plates. Ghosh & Bhattacharjee (2000) considered the effects of Hall current on a steady MHD fully developed flow in a rotating environment within a parallel plate channel in the presence of an inclined magnetic field. Ghosh (2002) considered a MHD couette flow in a rotating environment with non conducting walls in the presence of an arbitrary magnetic field. He considered how an interplay between the hydromagnetic force and coriolis force with an inclusion of Hall current affects the MHD flow behaviour. Denno & Fouad (1972) considered a nonuniform magnetic field on MHD channel flow between two parallel plates of infinite extent with the aim of reducing the amount of heat transfer from the fluid to the channel walls. Attia (2002) studied the unsteady flow and heat transfer of a dusty conducting fluid between two parallel plates with variable viscosity and electric conductivity. The fluid is driven by a constant pressure gradient and an external uniform magnetic field is applied perpendicular to the plates. The effect of the variation in the viscosity and electric conductivity of the fluid and the uniform magnetic field on the velocity and temperature fields for both the fluid and dust particles is discussed. Soundalgekar & Haldavnekar (1973) analyzed a fully-developed MHD free convective flow between two vertical, electrically conducting plates. They studied the effects of the external circuit, heat sources and modified boundary conditions on the temperature at the non-perfect thermally conducting plates. They observed that the flow is stable at small values of M, the Hartmann number, whereas at large values of M, an increase in the thermal conductance ratio or line heat source leads to an instability of the flow. However, they found that instability of the flow may be avoided by selecting the plates of high electrical conductivity. Seth & Jana (1980) investigated the unsteady hydromagnetic flow of a viscous incompressible fluid in the presence of a uniform transverse magnetic field in a rotating parallel plate channel with oscillating pressure gradient. An exact solution of the governing equations for the fully developed flow is obtained in closed form. They studied the effects of Hartmann number, the reciprocal of the Ekman number, and the frequency of oscillation on the flow field. They observed that for large values of the reciprocal of the Ekman number and frequency of oscillation there arises thin double-decker boundary layers near the plates of the channel and thin boundary layers for large Hartmann number. Bodosa & Borkakati (2003) analyzed the problem of an unsteady two-dimensional flow of a viscous incompressible and electrically conducting fluid between two parallel plates in the presence of a uniform transverse magnetic field when in case-I the plates are at different temperatures and in case-II the upper plate is considered to move with constant velocity whereas the lower plate is adiabatic. In most cases the Hall and ion-slip terms were ignored in applying Ohm's law, as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of MHD is towards a strong magnetic field, so that the influence of the electromagnetic force is noticeable (Cramer & Pai, 1973). Under these conditions, the Hall current and ion slip current are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Tani (1962) studied the Hall effect on the steady motion of electrically conducting and viscous fluids in channels. Soundalgekar *et al.* (1979); Soundalgekar & Uplekar (1986) studied the effect of the Hall currents on the steady MHD couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant (Soundalgekar & Uplekar, 1986) or to vary linearly along the plates in the direction of the flow (Soundalgekar *et al.*, 1979). Later, Attia (1998) extended the problem to the unsteady state with heat transfer, taking the Hall effect into consideration while neglecting ion-slip current. Nagy & Demendy (1995) investigated the influence of Hall currents and rotation on a generalized Hartmann flow and heat transfer. They considered a channel that is rotating with a constant angular velocity around an axis perpendicular to the walls in a uniform transverse magnetic field. The walls may have the same thickness, electrical and thermal conductivity as well as Hall parameter or different ones. The flow may be driven either by a pressure gradient or by motion of one of the walls. Exact solutions are derived for the velocity, magnetic field, viscous stress, current density, temperature distribution, yield components, electric current components and mean temperature as well as Nusselt numbers. Rao & Krishna (1982) analyzed the influence of Hall currents on the free and forced convective flow of a viscous rotating fluid between two horizontal plates. They discussed the effects of Hall currents on the velocity, the temperature and shear stress. Bharali & Borkakati (1982) studied the effect of Hall currents on MHD flow of an incompressible viscous electrically conducting fluid between two non-conducting porous plates in the presence of a strong uniform magnetic field. The flow is generated by a small uniform suction at the plates. Solutions are obtained for suction Reynolds number $Re << 1$, considering two cases for the imposed magnetic field, viz. (i) when the magnetic field is perpendicular to the plates, and (ii) when the magnetic field is parallel to the plates and perpendicular to the primary flow direction. The effect of the Hall currents on the flow as well as on the heat transfer is studied. They observed that in the absence of Hall currents, the change of the direction of the applied magnetic field does not affect the primary flow. Jana & Datta (1980) studied the combined effect of rotation and Hall current on the MHD couette flow. The heat transfer characteristic have also been discussed on taking the viscous and Joule dissipation into account. They observed that the primary and the secondary velocity components increase with increase in Hall parameter but the primary velocity decreases with increase in rotation parameter. Also, the rate of heat transfer at the stationary plate is independent of both the Hall parameter and the rotation parameter. The rate of heat transfer at the moving plate increases with increase in Hall parameter, while it decreases with increase in rotation parameter. The values of the critical Eckert number at which the direction of the heat flow changes increases with increase in Hall parameter. Hassanien & Mansour (1990) made an analysis of a two-dimensional unsteady flow of a viscous, incompressible electrically-conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field. The lower plate is fixed while the other one is oscillating in its own plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin-friction are derived. The effects of the magnetic parameters, permeability of the porous medium, and the frequency parameter are discussed. Krishna *et al.* (2002) made an initial value investigation of the hydromagnetic convection flow of a viscous electrically conducting fluid through a porous medium in a rotating parallel plate channel using boundary layer type equations. The exact solution of the governing equations is evaluated and the structure of the different boundary layers formed has been discussed. The ultimate quasi steady state velocity and temperature fields are numerically computed for various values of the governing parameters. Also the shear stress on the plates and the Nusselt numbers have been computed. Sarojamma & Krishna (1981) studied the combined free and forced convection in a rotating, viscous, incompressible fluid confined between two parallel porous plates. Assuming that the temperature varies linearly along the plates and the pressure gradient maintained uniform over the planes parallel to the plates, the velocity, temperature and the stresses are calculated analytically. Their behaviours for different values of the parameters viz., the Hartmann number, the Grashoff number and the suction parameter, are discussed graphically giving out the interplay between the various forces. Recently (Attia, 2005) studied the transient flow and heat transfer of an incompressible, viscous, electrically conducting fluid between two infinite non-conducting horizontal porous plates with the consideration of both Hall current and ion-slip current. The upper plate is moving with a constant velocity while the lower plate is kept stationary. The fluid is acted upon by a constant pressure gradient, a uniform suction and injection and a uniform magnetic field perpendicular to the plates. The induced magnetic field is neglected by assuming a very small magnetic Reynold's number (Sutton & Sherman, 1965), therefore, the uniform magnetic field is considered as the total magnetic field acting on the fluid. The two plates are maintained at two different but constant temperatures. The governing equations are solved numerically taking the Joule and the viscous dissipations into consideration in the energy equations. The effect of the magnetic field, Hall current, ion-slip, and suction and injection on both the velocity and temperature distributions is studied. Umavathi & Malashetty (2005) studied the laminar fully developed combined free and forced magnetoconvection in a vertical channel with symmetric and asymmetric boundary heating in the presence of viscous and Joulean dissipations.

1.3 Problem statement

In the studies given above, none of the researchers has focussed on the combined effects of rotation, heat transfer, mass transfer, porous media and Hall current in a MFD flow field. We endeavour to study a MFD fluid flow while considering the effects of rotation, Hall current, heat and mass transfer between two parallel porous, non conducting infinite plates.

1.4 Research objectives

This study aims at the following:

- Determining the velocity, temperature and concentration distributions for an electrically conducting fluid flow between two parallel plates in a rotating system.
- Investigating the effects of various fluid flow parameters on the velocity, temperature and concentration profiles for this fluid flow.

- Investigating the effects of some fluid flow parameters on the skin friction and the rates of heat and mass transfer. These parameters include the pressure gradient, hall parameter m, eckert number Ec, schmidt number Sc, suction parameter S , rotation parameter Er and time t.

1.5 Justification

The MFD flow between two parallel plates, has a solution that has many applications in MHD power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Mixed convection in vertical parallel plates finds applications in cooling of electronic devices and solar collectors. MHD flow through ducts has applications in design of MHD generators, cross field accelerators, shock tubes, pumps and flow meters. Other applications in engineering include designing of heat exchangers, electromagnetic pumps, space vehicle propulsion and braking and MHD electrical power generation. Fluid flows involving rotation are encountered in natural phenomena and find applications in the oceans and earth's atmosphere such as in meteorology and air pollution. It is in the light of this that we feel that this study will be of much interest.

The general equations governing a fluid flow which include the mass conservation equation, the momentum equation, the energy conservation equation and the Maxwell's equations are given in the next chapter. Later a fluid flow between two parallel straight plates is considered. The proposed method of solution to the equations that arise is also given.

Chapter 2

GOVERNING EQUATIONS AND METHODOLOGY

In this chapter, the equations governing the flow of an electrically conducting fluid in the presence of a magnetic field are given. We first state some approximations to be made when studying the flow of the electrically conducting fluid in the presence of a strong magnetic field. Thereafter, the basic conservation equations in hydrodynamics as well as electromagnetic theory are stated. The equations governing the flow of an electrically conducting fluid between two parallel plates in the presence of a magnetic field are also given. A mesh used to approximate the equations that govern the fluid flow to difference equations is given in this chapter. The methods that are later used to solve the resulting systems of equations are also given later in the chapter.

2.1 Approximations and assumptions

Some assumptions and approximations made to the fluid and to the fluid flow are given as follows.

- Laws of classical mechanics apply. The flow is limited to slow speed ($q<<c$) hence exclusion of relativistic effects.
- The length scale of the flow is taken to be large compared with the molecular mean free-path hence the fluid is taken as a continuum.
- The flow is incompressible where pressure variations don't produce any significant change in density.
- The fluid is Newtonian such that the fluids viscosity is assumed a constant.
- Fluid flow is unsteady.
- Fluid flow is laminar.

- There are no chemical reactions taking place in the fluid.
- There is no external electric field i.e. $\mathbf{E} = 0$.

2.1.1 Conservation equations

This study considers the flow of an electrically conducting fluid. On this basis, we give the general conservation equations in both hydrodynamics as well as electromagnetism. In the continuum approach of deriving the equations governing the motion of a fluid, the individual molecules are ignored and it is assumed that the fluid consists of continuous matter. This continuous matter which in this case is the fluid in consideration must satisfy the basic laws of mass conservation, momentum conservation, and conservation of energy. In the case where mass transfer is included, the equation of conservation of concentration has also to be considered. Fluid dynamics is concerned with the behaviour, subject to known laws, of a fluid of specified properties in a specified configuration. Maxwell's equations give the fundamentals of electricity and magnetism.

2.1.2 Continuity equation

The mass conservation equation also called the continuity equation is derived from the law of conservation of mass. Considering a section of the fluid flow region, the mass entering the section equals the mass leaving this section such that there is no mass being created or destroyed. For an unsteady fluid flow, the vector form of the continuity equation is derived in many fluid mechanics text books such as (Curie, 1974)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0 \tag{2.1}$$

where i = 1, 2, 3 along the x, y and z directions respectively. Since we are considering a fluid flow that is incompressible, the density of the fluid is assumed to be constant and in this case the continuity equation takes the form

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{2.2}$$

2.1.3 Momentum conservation equation

Conservation of momentum states that within some problem domain, the momentum remains constant. Momentum is neither created nor destroyed but only changed through the action of forces as described by Newton's laws of motion. Newton's second law of motion states that the rate of change of momentum of a body is equal to the net sum of resultant forces acting on the body. In this case, the body in consideration is the fluid. For a fluid in motion, some forces act within the fluid itself while other forces act on the fluid from external sources. Forces acting at a distance on a fluid particle are called body forces while surface forces refer to the forces due to direct contact of a fluid with other fluid particles or solid walls. The forces acting on the fluid have to be specified for a particular flow configuration being considered. The basic dynamical equation expressing Newton's second law of motion for a fluid of constant density is the Navier-Stokes equation given in tensor form as in (Curie, 1974).

$$\rho\frac{\partial u_j}{\partial t} + \rho u_k\frac{\partial u_j}{\partial x_k} = \frac{\partial \sigma_{i,j}}{\partial x_i} + \rho f_j \tag{2.3}$$

The forces considered are the force $\frac{\partial \sigma_{i,j}}{\partial x_i}$ due to surface shear stresses and the force ρf_j due to body forces acting on the fluid. The terms $\rho\frac{\partial u_j}{\partial t}$ and $\rho u_k\frac{\partial u_j}{\partial x_k}$ give the local acceleration and the convective acceleration respectively. The local acceleration represents the change in velocity with time at any point in space while the convective acceleration represents the change in velocity due to the fact that a given fluid element changes its position with time and therefore assumes different values of velocity as it flows. The most common type of body force which acts on a fluid is due to gravity. However, if we also include the electromagnetic forces then the total body force is $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_g$ where the electromagnetic force is given as in (Holman, 1992).

$$\mathbf{F}_e = \rho_e\mathbf{E} + \mathbf{J} \times \mathbf{B} \tag{2.4}$$

Since the force due to the applied electric field is negligible compared with the force due to the magnetic field, then the total electromagnetic force is given as

$$\mathbf{F}_e = \mathbf{J} \times \mathbf{B} \tag{2.5}$$

Taking the gravitational force per unit volume , the momentum conservation equation takes the form

$$\rho \frac{\partial u_j}{\partial t} + \rho u_k \frac{\partial u_j}{\partial x_k} = \frac{\partial \sigma_{ij}}{\partial x_i} - \rho g_j + (J \times B)_j \tag{2.6}$$

2.1.4 Equation of conservation of energy

The law of energy conservation is another fundamental law in Science which states that the energy in a given domain remains constant; it is neither created nor destroyed but can be converted from one form to another. In thermodynamics, energy and work of a system are related in the first law of thermodynamics which states that the change in the internal energy e is equal to the difference of the heat ΔQ transferred into a system and the work ΔW done by the system. i.e.

$$\Delta e = \Delta Q - \Delta W \tag{2.7}$$

This equation is derived in most standard fluid mechanics textbooks as

$$\rho \frac{De}{Dt} = -\nabla Q + Q' - P\nabla \cdot \mathbf{q} + \mu\phi \tag{2.8}$$

In three dimensions, the viscous dissipation function is expressed in Cartesian co-ordinates as follows

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2 \tag{2.9}$$

The thermodynamic definition of the enthalpy

$$h = E + \left(\frac{1}{\rho}\right) P \tag{2.10}$$

is given in differential form as

$$dh = dE + \left(\frac{1}{\rho}\right) dP + Pd\left(\frac{1}{\rho}\right) \tag{2.11}$$

Using Maxwell's thermodynamic relation

$$dE = TdS - Pd\left(\frac{1}{\rho}\right) \tag{2.12}$$

in equation (2.11), we have

$$dh = TdS + \frac{1}{\rho}dP \tag{2.13}$$

Taking the entropy to be a function of pressure and temperature, i.e. S = S (P,T), we have

$$dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial P}\right)_T dP \tag{2.14}$$

and using the generalised thermodynamic relations

$$\begin{aligned} \left(\tfrac{\partial S}{\partial P}\right)_T &= \tfrac{-\beta}{\rho} \\ \left(\tfrac{\partial S}{\partial T}\right)_P &= \tfrac{C_p}{T} \end{aligned} \tag{2.15}$$

the equation (2.14) reduces to

$$dS = \frac{C_p}{T}dT - \frac{\beta}{\rho}dP \tag{2.16}$$

Substituting equation (2.16) in equation (2.13) we have

$$dh = C_p dT + \frac{1}{\rho}\left(1 - \beta T\right) dP \tag{2.17}$$

Using Fourier's law of heat conduction we have

$$Q = -K\nabla T \tag{2.18}$$

On substituting equation (2.18) in equation (2.8) the energy equation can be written as

$$\rho C_p \frac{DT}{Dt} = K\nabla^2 T + Q' + \beta T \frac{Dp}{Dt} + \mu\phi \tag{2.19}$$

The dissipation function Q$'$ for the hydromagnetic flow in consideration can be taken to be a result of electromagnetic interactions. The electrical dissipation which is the heat energy produced by the work done by electrical current is equal to **J.E** . This dissipative heat due to electric currents is $\frac{j^2}{\sigma}$, which is the Joule heating. Considering constant physical properties, equation (2.19) thus becomes

$$\rho C_p \frac{DT}{Dt} = K\nabla^2 T + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)^2 + \frac{1}{\sigma}j^2 \tag{2.20}$$

2.1.5 Concentration equation

The concentration equation is also called the diffusion equation and is based on the principle of mass conservation for each component or constituent in a fluid mixture. In the absence of chemical reactions, the rate at which the mass of some species enters a control volume minus the rate at which the species mass leaves the control volume is equal to the rate at which the species mass is stored in the control volume. The species could be some substance mixed in, dissolved in, or otherwise carried by the fluid. This occurs in air pollution where there are pollutants and in chemical engineering processes which involve mixing of different substances among other areas. The distribution of the concentration C (mass of the substance per unit volume) is determined by its advection by moving fluid particles and by its diffusion between fluid particles. In this case we have the concentration equation given as

$$\frac{DC}{Dt} = K_c \nabla^2 C \tag{2.21}$$

where K_c is the diffusion coefficient depending on both the fluid and on the diffusing substance.

2.1.6 Maxwell's equations

These are the basic equations in electricity and magnetism and they give relations between the interacting electric and magnetic fields. These are given in many books on electromagnetic theory such as (Moreau, 1990). In the absence of magnetic or polarized media these equations are stated in differential form in the laws below.

Gauss law for electricity

This law states that the net flux of electric field lines out of a closed surface S is proportional to the net charge enclosed within the surface.

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon_0} = 4\pi k \rho \tag{2.22}$$

Gauss' law for magnetism

This law states that all magnetic fields **B** have field lines that are continuous.

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{2.23}$$

Faraday's law of induction

This law states that changing magnetic fields produce an electric field. Here the e.m.f induced in a circuit is equal to the rate of change with time of the total magnetic flux through the circuit no matter how the flux changes.

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.24}$$

Ampere's law

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{4\pi k}{c^2}\mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \tag{2.25}$$

Here, k is Coulomb's constant given as $k = \frac{1}{4\pi\varepsilon_0} \quad and \quad c^2 = \frac{1}{\mu_0\varepsilon_0}$, where **E** is the electric field (N/C), **B** is the magnetic field (T), ρ_e is the charge density, ε_0 is the electric permittivity of free space (F/m), μ_0 the magnetic permeability of free space (N/A^2), J the current density and c is the speed of light (m/s). The differential form of Maxwell's equations with magnetic and/or polarized media are given in any standard text book in electromagnetism as stated below.

Gauss' law for electricity

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_e$$

$$\left.\begin{array}{l} \mathbf{D} = \varepsilon_0\mathbf{E} + \rho \\ \mathbf{D} = \varepsilon_0\mathbf{E} \\ \mathbf{D} = \varepsilon\mathbf{E} \end{array}\right\} \tag{2.26}$$

The electric displacement $\boldsymbol{D}$ is given in equations (2.26) respectively for the general case, free space and isotropic linear dielectric.

Gauss' law for magnetism

$$\boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{2.27}$$

Faraday's law of induction

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{2.28}$$

Ampere's law

$$\boldsymbol{\nabla} \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \tag{2.29}$$

Here $\frac{\partial \mathbf{D}}{\partial t}$ is the displacement current which is included when currents change with time and current can pile up electric charges. In a good conductor, there is a high rate of charge removal and the term can be dropped to give

$$\nabla \times \mathbf{H} = \mathbf{J} \tag{2.30}$$

where

$$\left.\begin{array}{l}\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \\ \mathbf{B} = \mu_0\mathbf{H} \\ \mathbf{B} = \mu_e\mathbf{H}\end{array}\right\} \tag{2.31}$$

The magnetic field **B** given in equations (2.31) represent the general case, free space and isotropic linear magnetic medium respectively.

2.1.7 Lorentz's force law

This law defines the total force resulting from both the electric and magnetic fields. When a small test charge Q is placed in an electric field **E**, it experiences a force $\mathbf{F}_e$ given as

$$\boldsymbol{F}_e = Q\boldsymbol{E} \tag{2.32}$$

The presence of a magnetic field makes the test charge experience a magnetic force $\mathbf{F}_m$. This force $\mathbf{F}_m$ acts at right angles to the velocity vector **v** of the test charge as well as the magnetic flux density **B**, and in this case we have

$$\mathbf{F}_m = Q\mathbf{v} \times \mathbf{B} \tag{2.33}$$

The total electromagnetic force which is the Lorentz force on the test charge Q is then given as

$$\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m = Q\mathbf{E} + Q\mathbf{v} \times \mathbf{B} \tag{2.34}$$

where Q**E** is the electric force and Q$\mathbf{v} \times \mathbf{B}$ is the magnetic force. The magnetic field strength $\mathbf{H} = \frac{\mathbf{B}_0}{\mu_e}$

2.1.8 Charge conservation

The principle of electric charge conservation is a fundamental law that states that charge is conserved and thus cannot be created nor destroyed. This fundamental idea of charge conservation is contained in Maxwell's equations. Taking the divergence of the differential form of Amperes law, we have

$$\nabla \cdot (\nabla \times \mathbf{B}) = \frac{\nabla \cdot \mathbf{J}}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial}{\partial t}(\nabla \cdot \mathbf{E}) = 0 \tag{2.35}$$

Using Gauss law $\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$ in equation (2.35) we have

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \tag{2.36}$$

This implies that the current through any enclosed surface is equal to the time rate of change of charge density within the surface. For steady currents, the charge density does not vary with time and we have

$$\nabla \cdot \mathbf{J} = 0 \tag{2.37}$$

2.1.9 Equation of electric current density

The electric current density for a substance having conductivity σ is given as

$$\mathbf{J} = \sigma \mathbf{E}' \tag{2.38}$$

In this case $\mathbf{E}'$ is the electric field experienced by a fluid element in its rest frame. However if a fluid is moving with velocity $\boldsymbol{q}$ with respect to an applied magnetic field $\mathbf{B}$ an electric field arises due to this motion and is described by the Lorentz transformation

$$\mathbf{E}' = \mathbf{E} + \mathbf{q} \times \mathbf{B} \tag{2.39}$$

Here $\mathbf{E}$ is the electric field in the laboratory frame and higher order relativistic conditions have been neglected since $q^2 << c^2$. Substituting equation (2.39) in equation (2.38) we have

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{q} \times \mathbf{B}) \tag{2.40}$$

Equation (2.40) is referred to by Dendy (1990) as the generalized Ohm's law.

2.2 MHD fluid flow between parallel plates with constant suction and injection

2.2.1 Introduction

An electrically conducting fluid flowing between two parallel porous plates is considered in this section. The equations governing an electrically conducting fluid flow in the presence of magnetic field are presented in dimensional form. Later in the chapter, by the use of scaling variables and non-dimensional parameters the governing equations are presented in non-dimensional form. The approximations stated at the beginning of this chapter also apply here. However, we have the following additional approximations.

- The porous plates have constant suction and injection.
- The electric force $\rho_e \mathbf{E}$ due to the electric field is negligible compared with the force due to the magnetic field.
- The flow induced magnetic field is negligible which is justified for very small magnetic Reynold's number (Shercriff, 1965).
- The fluid has constant electrical conductivity and constant thermal conductivity.
- The fluid is considered to be electrically neutral with no surplus electric charge distribution. This is because the space charge transport is negligible compared with J for a good conductor.
- The Boussinesq approximation is considered such that:
  - All transport properties except density are treated as constants.
  - The variation of density is negligible except when it directly causes buoyancy forces.
  - Density varies linearly with temperature and concentration and the deviation from a reference value is small.

We consider an unsteady, laminar, viscous fully developed fluid flowing between two long horizontal porous parallel plates and later consider flow between two vertical porous plates.

2.2.2 Fluid flow between two horizontal parallel plates

Let the two electrically non conducting horizontal plates be located at the planes $y = \pm h$ with the planes extending infinitely on $x \in (-\infty, \infty)$ and $z \in (-\infty, \infty)$. The fluid flows between the two plates under the influence of a constant pressure gradient in the x direction, and a uniform suction from above and injection from below with constant velocity v_0 which are all applied at time t = 0. The system is subjected to a uniform magnetic field B_0 which is perpendicular to the main flow. It is assumed that the induced magnetic field is negligible which is justified for very small magnetic Reynold's number. Therefore the uniform magnetic field B_0 is considered as the total magnetic field acting on the fluid. The Cartesian co ordinate system is chosen with the transverse coordinate Y which is the direction of the magnetic field i.e. $\mathbf{B}_0 = \langle 0, B_0, 0 \rangle$ and the coordinate in the direction of the fluid flow is X which is also parallel to the plates as shown in figure 2.1 below.

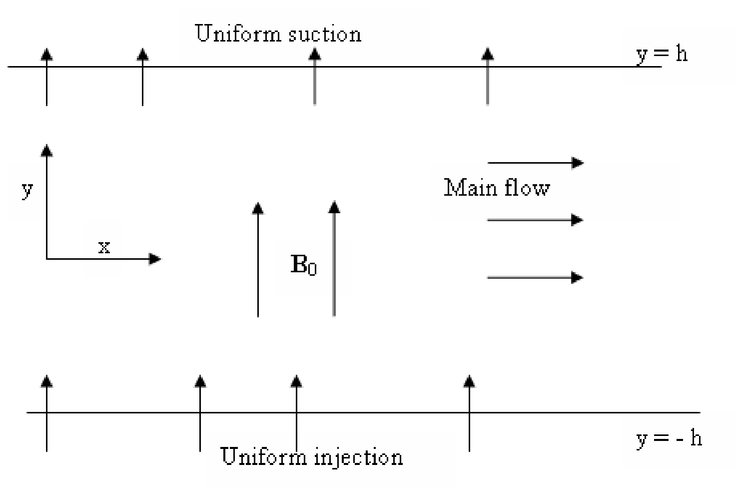

FIGURE 2.1. Fluid flow between two parallel horizontal plates

(a) Mathematical formulation

From the geometry of the problem, all quantities except the pressure gradient $\frac{dP}{dx}$, which is assumed constant, do not depend on the coordinates X and Z. The velocity vector of the fluid is $\boldsymbol{q} = \boldsymbol{q}$(y,t). Considering Ohm's law in the form $\mathbf{J} = \sigma(\mathbf{q} \times \mathbf{B})$ we have

$$\begin{aligned} \mathbf{J} &= \sigma[(u\mathbf{i} + v_0\mathbf{j}) \times B_0\mathbf{j}] \\ &= \sigma B_0 u\mathbf{k} \end{aligned} \tag{2.41}$$

$$\begin{aligned} \mathbf{J} \times \mathbf{B} &= \sigma B_0 u\mathbf{k} \times B_0\mathbf{j} \\ &= -\sigma B_0^2 u\mathbf{i} \end{aligned} \tag{2.42}$$

Since $B_0 = \mu_e H_0$ then $\mathbf{J} \times \mathbf{B} = -\sigma\mu_e^2 H_0^2 u\mathbf{i}$

or in component form

$$(\mathbf{J} \times \mathbf{B})_x = -\sigma\mu_e^2 H_0^2 u \tag{2.43}$$

Since $\mathbf{q} = \mathbf{q}$ (y,t) the viscous term $\mu\nabla^2\boldsymbol{q}$ takes the form $\mu\frac{\partial^2 u}{\partial y^2}$. And substituting these in the momentum equation (2.6) and considering the electromagnetic force terms in equation (2.43), we have

$$\rho\left(\frac{\partial u^*}{\partial t^*} + v_0^*\frac{\partial u^*}{\partial y^*}\right) = -\frac{dP^*}{dx^*} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - \sigma\mu_e^2 H_o^2 u^* \tag{2.44}$$

By taking into account the effect of viscous dissipation the energy equation (2.20) takes the form

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*} + v_0^*\frac{\partial T^*}{\partial y^*}\right) = K\frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left(\frac{\partial u^*}{\partial y^*}\right)^2 \tag{2.45}$$

The equation (2.21) of concentration conservation can be written as

$$\frac{\partial C^*}{\partial t^*} + v_0^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{2.46}$$

(b) General scaling variables

In this study the non-dimensionalisation process is based on the following sets of general scaling variables and the non-dimensional parameters.

$$\begin{gathered} t = \frac{t^* U_0^2}{v}, \quad x = \frac{x^* U_0}{v}, \quad y = \frac{y^* U_0}{v}, \quad z = \frac{z^* U_0}{v}, \\ u = \frac{u^*}{U_0}, \quad v = \frac{v^*}{U_0}, \quad P = \frac{P^*}{\rho U_0^2} \\ \theta = \frac{T^* - T_1^*}{T_2^* - T_1^*}, \quad C = \frac{C^* - C_1^*}{C_2^* - C_1^*} \end{gathered} \tag{2.47}$$

The star (*) superscript has been used to denote the dimensional variables.

(c) Non-dimensional parameters

The parameters considered in this chapter include parameters in both hydrodynamics as well as other parameters due to the interaction with the electromagnetic field.

Pressure Parameter R_p

$$R_p = \frac{P_o}{\rho U_o^2}$$

For incompressible flow the pressure number is usually of order unity.

Mach Number

This is given as the ratio of the flow velocity to the velocity of sound i.e.

$$\frac{U}{a_0}$$

Reynold's Number Re

This is the ratio of viscous to inertia forces

$$Re = \frac{\rho U_0 h}{\mu} = \frac{U_0 h}{\upsilon}$$

Prandtl Number Pr

$$\Pr = \frac{\upsilon \rho C_p}{K} = \frac{\mu C_p}{K}$$

This represents the ratio of the momentum diffusion to thermal diffusivity K. Pr thus provide a measure of relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers respectively.

Grashof Number Gr

$$Gr = \frac{\upsilon g \beta \left[T_2^* - T_1^*\right]}{U_0^3}$$

This defines a measure of the ratio of buoyancy to viscous forces in the velocity boundary layer.

Ekcert Number Ec

$$Ec = \frac{U_0^2}{C_p \left(T_2^* - T_1^*\right)}$$

This is a measure of Kinetic energy of the flow relative to the enthalpy differences across the boundary layer.

Magnetic Parameter M_1

$$M_1 = \sqrt{\frac{\sigma \mu_e^2 H_0^2 \upsilon}{\rho U_0^2}}$$

This is the ratio of the magnetic force to the inertia force.

Schmidt Number Sc

$$Sc = \frac{\upsilon}{D}$$

This provides a measure of the relative effectiveness of the momentum and mass transport by diffusion in the velocity and concentration boundary layers respectively. For convection mass transfer in laminar flows, it determines the relative velocity and concentration boundary layer thickness.

Suction Parameter

$$S = \frac{v_0^*}{U_0}$$

This is a ratio of the suction velocity to the velocity of the plate.

(d) The non-dimensionalisation process

The process of non-dimensionalisation allows us to apply results obtained for a surface experiencing a set of conditions to a geometrically similar surface which may be experiencing entirely different conditions. These conditions may vary, for example with the nature of the fluid, the fluid velocity and/or with the size of the surface. If the non dimensional parameters are the same for two geometrically similar situations, then the equations of the non dimensional variables are the same. Hence, they have the same solutions and the same flow patterns. We use the scaling variables and non-dimensional parameters quoted above to non-dimensionalize the equations governing the fluid flow under consideration. The terms in the momentum conservation equation (2.44) can be non-dimensionalized as follows:

$$\frac{\partial u^*}{\partial t^*} = \frac{U_0}{\upsilon/U_0^2}\frac{\partial u}{\partial t} = \frac{U_0^3}{\upsilon}\frac{\partial u}{\partial t}$$

$$\frac{\partial u^*}{\partial y^*} = \frac{U_0}{\upsilon/U_0}\frac{\partial u}{\partial y} = \frac{U_0^2}{\upsilon}\frac{\partial u}{\partial y}$$

$$\upsilon_0^*\frac{\partial u^*}{\partial y^*} = S\frac{U_0^3}{\upsilon}\frac{\partial u}{\partial y}$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{\partial}{\partial y^*}\left(\frac{U_0^2}{\upsilon}\frac{\partial u}{\partial y}\right) = \frac{U_0^3}{\upsilon^2}\frac{\partial^2 u}{\partial y^2}$$

$$\upsilon\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{U_0^3}{\upsilon}\frac{\partial^2 u}{\partial y^2}$$

$$\frac{dP^*}{dx^*} = \frac{d\left(\rho P U_0^2\right)}{d\left(\frac{\upsilon x}{U_0}\right)} = \frac{\rho U_0^2}{\frac{\upsilon}{U_0}}\frac{dP}{dx} = \frac{\rho U_0^3}{\upsilon}\frac{dP}{dx}$$

$$\frac{1}{\rho}\frac{dP^*}{dx^*} = \frac{U_0^3}{\upsilon}\frac{dP}{dx}$$

$$\frac{\sigma\mu_e^2 H_0^2 u^*}{\rho} = \frac{\sigma\mu_e^2 H_0^2 U_0 u}{\rho}$$

Dividing each of the terms given above by $\frac{U_0^3}{\upsilon}$, the momentum equation (2.44) becomes

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma\mu_e^2 H_0^2 \upsilon u}{\rho U_0^2} \tag{2.48}$$

On introducing the non-dimensional parameter M^2, equation (2.48) take the form

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu\frac{\partial^2 u}{\partial y^2} - M^2 u \tag{2.49}$$

Considering each of the terms in the equation of energy (2.45), we have the transformations

$$\frac{\partial T^*}{\partial t^*} = \frac{(T_2^* - T_1^*)}{\upsilon / U_0^2} \frac{\partial \theta}{\partial t} = \frac{U_0^2 (T_2^* - T_1^*)}{\upsilon} \frac{\partial \theta}{\partial t}$$

$$\frac{\partial T^*}{\partial y^*} = \frac{(T_2^* - T_1^*)}{\upsilon / U_0} \frac{\partial \theta}{\partial y} = \frac{U_0 (T_2^* - T_1^*)}{\upsilon} \frac{\partial \theta}{\partial y}$$

$$\upsilon_0^* \frac{\partial T^*}{\partial y^*} = \frac{U_0^2 (T_2^* - T_1^*)}{\upsilon} S \frac{\partial \theta}{\partial y}$$

$$\frac{\partial^2 T^*}{\partial y^{*2}} = \frac{U_0 (T_2^* - T_1^*)}{\upsilon} \frac{\partial}{\partial y^*} \left(\frac{\partial \theta}{\partial y} \right) = \frac{U_0^2}{\upsilon^2} (T_2^* - T_1^*) \frac{\partial^2 \theta}{\partial y^2}$$

$$\frac{\partial u^*}{\partial y^*} = \frac{U_0^2}{\upsilon} \frac{\partial u}{\partial y}$$

$$\left(\frac{\partial u^*}{\partial y^*} \right)^2 = \frac{U_0^4}{\upsilon^2} \left(\frac{\partial u}{\partial y} \right)^2$$

If each of the terms given above is multiplied by $\frac{\upsilon}{U_0^2 \left(T_2^* - T_1^*\right)}$ we will have the energy equation in the form

$$\frac{\partial \theta}{\partial t} + S \frac{\partial \theta}{\partial y} = \frac{K}{\upsilon \rho C_P} \frac{\partial^2 \theta}{\partial y^2} + \frac{\mu U_0^2}{\upsilon \rho C_P (T_2^* - T_1^*)} \left(\frac{\partial u}{\partial y} \right)^2 \tag{2.50}$$

Using non-dimensional parameters Pr, Ec and S, equation (2.50) becomes

$$\frac{\partial \theta}{\partial t} + S \frac{\partial \theta}{\partial y} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial u}{\partial y} \right)^2 \tag{2.51}$$

The components of the concentration equation (2.46) can be transformed as shown below

$$\frac{\partial C^*}{\partial t^*} = \frac{(C_2^* - C_1^*)}{\upsilon / U_0^2} \frac{\partial C}{\partial t} = \frac{U_0^2 (C_2^* - C_1^*)}{\upsilon} \frac{\partial C}{\partial t}$$

$$\upsilon_0^* \frac{\partial C^*}{\partial y^*} = \frac{(C_2^* - C_1^*)}{\upsilon / U_0^2} S \frac{\partial C}{\partial y} = \frac{U_0^2 (C_2^* - C_1^*)}{\upsilon} S \frac{\partial C}{\partial y}$$

$$\frac{\partial^2 C^*}{\partial y^{*2}} = \frac{U_0 (C_2^* - C_1^*)}{\upsilon} \frac{\partial}{\partial y^*} \left(\frac{\partial C}{\partial y} \right) = \frac{U_0^2 (C_2^* - C_1^*)}{\upsilon^2} \frac{\partial^2 C}{\partial y^2}$$

Multiplying each term by $\frac{v}{U_0^2\left(C_2^*-C_1^*\right)}$, the concentration equation takes the form

$$\frac{\partial C}{\partial t}+S\frac{\partial C}{\partial y}=\frac{D}{v}\frac{\partial^2 C}{\partial y^2} \tag{2.52}$$

and using the non-dimensional parameter Sc ,equation (2.52) becomes

$$\frac{\partial C}{\partial t}+S\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{2.53}$$

Equations (2.49), (2.51) and (2.53) respectively give the final set of conservation of momentum, energy and concentration equations in non-dimensional form.

2.2.3 Fluid flow between two vertical parallel plates

Consider an unsteady, laminar electrically conducting and fully developed viscous fluid flowing between two parallel vertical plates as shown in figure 2.2. The

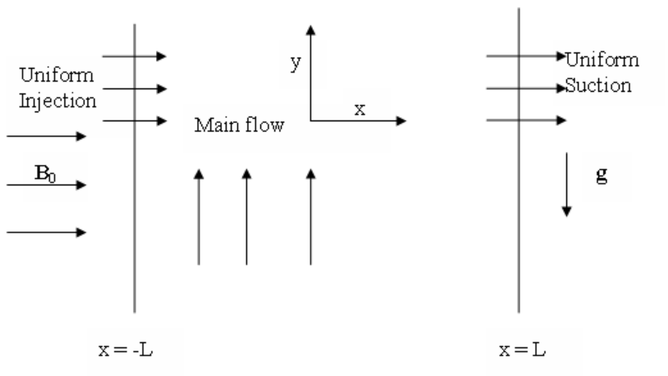

FIGURE 2.2. Fluid flow between two vertical parallel plates

Cartesian coordinate system is chosen such that the flow direction is parallel to the vertical y axis. A constant magnetic field of strength B_0 is applied across the plates in the x direction. The walls are chosen relative to the origin of the axes such that the plates are on the planes $x=-L$ and $x=L$. These plates are taken to be electrically non-conducting. The fluid flows in the positive y direction with a velocity U. Since the flow is fully developed, then

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.54}$$

Considering a uniform injection of a second material from the left and uniform suction to the right with velocity u_0 we have from $\frac{\partial u}{\partial x} = 0$ so that

$$u = u_0 \tag{2.55}$$

Taking Ohm's law in the form $\mathbf{J} = \sigma(\mathbf{q} \times \mathbf{B})$ where $\mathbf{q} = u_0\mathbf{i} + v(x,t)\mathbf{j}$ and $\boldsymbol{B} = B_0\mathbf{i}$ we have

$$\begin{aligned}(\mathbf{J} \times \mathbf{B}) &= [\sigma(u_0\mathbf{i} + v\mathbf{j}) \times B_0\mathbf{i}] \times B_0\mathbf{i} \\ &= -\sigma B_0^2 v\mathbf{j} \\ &= -\sigma\mu_e^2 H_0^2 v\mathbf{j}\end{aligned}$$

We consider a slow speed fluid flow such that the buoyancy force resulting from temperature and concentration differences in the flow field are comparable with the inertia and viscous forces. In the presence of heat transfer, let the density vary with temperature and also vary with concentration difference in the presence of mass transfer. Using the Boussinesq approximation, let the thermodynamic state of a fluid depend on the pressure, temperature and concentration and if we consider small density variations at constant pressure we have

$$\rho \cong \rho_\infty + \left(\frac{\partial \rho}{\partial T^*}\right)_P (T^* - T_1^*) + \left(\frac{\partial \rho}{\partial C^*}\right)_P (C^* - C_1^*) \tag{2.56}$$

T_1^* and C_1^* are the reference temperature and concentration respectively.

Using

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T^*}\right)_P \; and$$

$$\beta_c = \frac{-1}{\rho}\left(\frac{\partial \rho}{\partial C^*}\right)_P \tag{2.57}$$

in equation (2.56) we have

$$\rho_\infty - \rho = \rho\beta\left(T^{*-}T_1^*\right) + \rho\beta_c\left(C^* - C_1^*\right) \tag{2.58}$$

To determine the pressure gradient term the momentum equation is evaluated at the edge of the boundary layer where $\rho \to \rho_\infty$. A pressure gradient will exist in the y^* direction due to the change in elevation and we have the pressure gradient due to fluid of density ρ_∞ given as

$$\frac{\partial P}{\partial y^*} = -\rho_\infty g \tag{2.59}$$

The body force term in the momentum conservation equation along the y^* direction i.e.$-\nabla P^* - \rho g$

gives

$$-\frac{\partial p}{\partial y^*} - \rho g \tag{2.60}$$

Substituting equations (2.59) in equation (2.60) and using equation (2.58) we have

$$-\rho g - \frac{\partial p}{\partial y^*} = -\rho g + \rho_\infty g = \rho g \left[\beta \left(T^{*-} T_1^*\right) + \beta_c \left(C^* - C_1^*\right)\right] \tag{2.61}$$

The momentum, energy and concentration conservation equations respectively take the dimensional forms

$$\rho \left(\frac{\partial v^*}{\partial t^*} + u_0^* \frac{\partial v^*}{\partial x^*}\right) = \mu \frac{\partial^2 v^*}{\partial x^{*2}} + \rho g \left[\beta \left(T^* - T_1^*\right) + \beta_c \left(C^* - C_1^*\right)\right] - \sigma \mu_e^2 H_o^2 v^* \tag{2.62}$$

$$\rho C_p \left(\frac{\partial T^*}{\partial t^*} + u_0^* \frac{\partial T^*}{\partial x^*}\right) = K \frac{\partial^2 T^*}{\partial x^{*2}} + \mu \left(\frac{\partial u^*}{\partial x^*}\right)^2 \tag{2.63}$$

$$\frac{\partial C^*}{\partial t^*} + u_0^* \frac{\partial C^*}{\partial x^*} = D \frac{\partial^2 C^*}{\partial x^{*2}} \tag{2.64}$$

These equations governing the fluid flow can be represented in non dimensional form

$$\frac{\partial v}{\partial t} + S \frac{\partial v}{\partial x} = \frac{\partial^2 v}{\partial x^2} + Gr\theta + GcC - M^2 v \tag{2.65}$$

$$\frac{\partial \theta}{\partial t} + S \frac{\partial \theta}{\partial x} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial x^2} + Ec \left(\frac{\partial u}{\partial x}\right)^2 \tag{2.66}$$

$$\frac{\partial C}{\partial t} + S \frac{\partial C}{\partial x} = \frac{1}{Sc} \frac{\partial^2 C}{\partial x^2} \tag{2.67}$$

2.3 Methodology

The proposed method of solving the system of non linear equations which arise from the flow model is the numerical approximation method of finite differences. To compute the skin friction, the rate of heat transfer and the rate of mass transfer we apply the method of least squares. These methods are discussed in the sections that follow. To be able to describe these numerical approximation methods we first define a flow mesh.

2.3.1 Definition of mesh

We want to use a uniform mesh to represent a function of two variables $f(y,t)$ where y is the distance and t is the time. Consider a yt-plane which is divided into uniform rectangular cells of width Δy and height Δt as shown in figure 2.3. Consider a reference point (i,j) where i and j represent y and t respectively. Using the notation $i\pm1$ for $y\pm\Delta y$ and $j\pm1$ for $t\pm\Delta t$ we can define the adjacent points to y and t, the points that are i and j units from the reference point will have coordinates $(i\Delta y, j\Delta t)$.

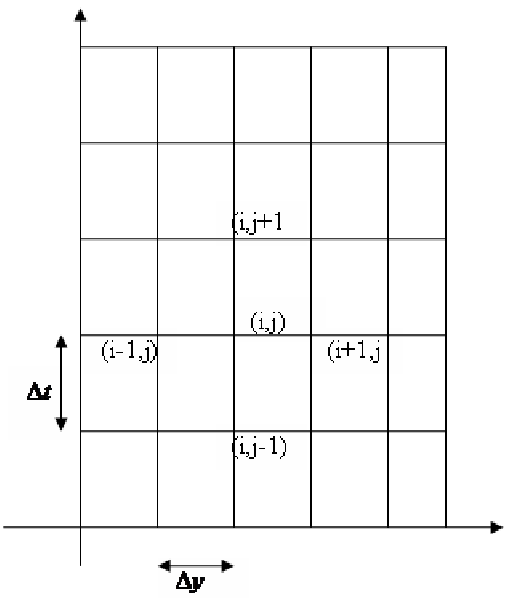

FIGURE 2.3. Mesh representing flow

2.3.2 The finite difference method

The finite difference approximation amounts to replacing derivatives with finite differences. Considering a function $f = f(y)$ which is continuous around $y = b$ with $a = b - \Delta y$ and $c = b + \Delta y$. The first derivative $f'(y)$ of the function $f(y)$ at the point b is approximated by finite differences in one of three ways called the central, backward and forward difference formulae as given below respectively as

$$\frac{df}{dy} \approx \frac{\delta f}{\Delta y} = \frac{f(c)-f(a)}{2\Delta y}$$

$$\frac{df}{dy} \approx \frac{\nabla f}{\Delta y} = \frac{f(b)-f(a)}{\Delta y}$$

$$\frac{df}{dy} \approx \frac{\Delta f}{\Delta y} = \frac{f(c)-f(b)}{\Delta y}$$

As the value of Δy becomes smaller the differences become better estimates. When the function depends on more than one variable, the finite differences can as well be used to approximate the partial derivatives. In particular for a function of two variables $f(y, t)$ we have the first central, backward and forward differences at the point (b, t) given respectively as

$$\frac{\partial f}{\partial y} \approx \frac{\delta f}{\Delta y} = \frac{f(b+\Delta y,t)-f(b-\Delta y,t)}{2\Delta y} \qquad \frac{\partial f}{\partial t} \approx \frac{\delta f}{\Delta t} = \frac{f(b,t+\Delta t)-f(b,t-\Delta t)}{2\Delta t}$$

$$\frac{\partial f}{\partial y} \approx \frac{\nabla f}{\Delta y} = \frac{f(b,t)-f(b-\Delta y,t)}{\Delta y} \qquad \frac{\partial f}{\partial t} \approx \frac{\nabla f}{\Delta t} = \frac{f(b,t)-f(b,t-\Delta t)}{\Delta t}$$

$$\frac{\partial f}{\partial y} \approx \frac{\Delta f}{\Delta y} = \frac{f(b+\Delta y,t)-f(b,t)}{\Delta y} \qquad \frac{\partial f}{\partial t} \approx \frac{\Delta f}{\Delta t} = \frac{f(b,t+\Delta t)-f(b,t)}{\Delta t}$$

In our solution, we will use the forward differences. Higher order finite differences are similarly given such that the second order forward differences in y and t are respectively of the form

$$\frac{\partial^2 f}{\partial y^2} \approx \frac{\Delta^2 f}{(\Delta y)^2} = \frac{f(b + \Delta y, t) - 2f(b, t) + f(b - \Delta y, t)}{(\Delta y)^2}$$

and

$$\frac{\partial^2 f}{\partial t^2} \approx \frac{\Delta^2 f}{(\Delta t)^2} = \frac{f(b, t + \Delta t) - 2f(b, t) + f(b, t - \Delta t)}{(\Delta t)^2}$$

2.3.3 The least squares approximation method

We use the Least Squares approximating method to calculate values of the rate of mass transfer, the skin friction and the rate of heat transfer at each of the boundaries. The least squares approximation method is a convenient procedure for determining best approximations. The procedure is to seek a polynomial that is as close as possible to a function that could be used to represent the given flow model. The Least Squares method aims at determining the approximating function such that we minimize the error between the finite difference approximation values and the actual functional values of the variables. This approximating function is then used to compute the values of the rate of mass transfer, the skin friction and the rate of heat transfer at each of the plates.The rate of mass transfer at the plate is given by (Incropera & Dewitt, 1985).

$$Sh = -\left.\frac{\partial C}{\partial y}\right|_{y=l} \tag{2.68}$$

The skin friction is the average rate of shear stress at the plates due to velocity and it is given in the x and z axes respectively as (Incropera & Dewitt, 1985).

$$\tau_x = -\left.\frac{\partial u}{\partial y}\right|_{y=l} \tag{2.69}$$

and

$$\tau_z = -\left.\frac{\partial v}{\partial y}\right|_{y=l} \tag{2.70}$$

The rate of heat transfer on the other hand is given by (Incropera & Dewitt, 1985).

$$Nu = -\left.\frac{\partial \theta}{\partial y}\right|_{y=l} \tag{2.71}$$

The second order Least Squares correlation will be used to calculate the values of the rate of mass transfer Sh, the skin friction in the direction of the primary velocity, the skin friction in the direction of the secondary velocity and the rate of heat transfer Nu at each of the boundaries. For this study we consider a quadratic bivariate polynomial which is a function of distance (y) and time (t). The second degree bivariate polynomials approximating the primary velocity u, secondary velocity w, temperature θ , and concentration C in the variables y and t are given by

$u'(y,t) = a_1 + b_1 y + c_1 t + d_1 y^2 + e_1 t^2 + f_1 yt$

$v'(y,t) = a_2 + b_2 y + c_2 t + d_2 y^2 + e_2 t^2 + f_2 yt$

$\theta'(y,t) = a_3 + b_3 y + c_3 t + d_3 y^2 + e_3 t^2 + f_3 yt$

$C'(y,t) = a_4 + b_4 y + c_4 t + d_4 y^2 + e_4 t^2 + f_4 yt$

The Least Squares method aims at determining the constants $a_i, b_i, c_i, ...f_i$ where $i = 1, 2, 3, 4$ such that we minimize the error between the finite difference approximation values and the actual functional values of the variables. We thus seek to minimize

$$I(a_i, b_i, c_i, ..., f_i) = \sum_{j=0}^{N} \left[X\left(y_j, t_j\right) - X'\left(y_j, t_j\right)\right]^2 \qquad (2.72)$$

In this equation, $X(y_j, t_j)$ and $X'(y_j, t_j)$ are respectively the exact and approximate values of the primary and secondary velocities, temperature and concentration. The condition of minimizing the equation (2.72) is satisfied if the normal equations given below are satisfied.

$$\frac{\partial I}{\partial a_i} = \frac{\partial I}{\partial b_i} = ... = \frac{\partial I}{\partial f_i} = 0 \qquad (2.73)$$

i = 1,2,3,4. The four systems of equations that result from the normal equations (2.73) for each i, are each solved for the constants $a_i, b_i, ..., f_i$ over the first five points.

In the next chapter, we study the combined effects of heat transfer, mass transfer, porous media and Hall current in a MHD flow field between parallel porous plates in the presence of a strong magnetic field.

Chapter 3

MHD fluid flow between parallel porous plates with effect of Hall current

In chapter two, we considered the general case in applying Ohm's law on the fluid flow problem. The approximation where the Hall current is ignored is acceptable in cases where the applied magnetic field is small or is of moderate value. When a strong magnetic field is considered however, the effect of the electromagnetic force is significant and has to be considered in the analysis of the fluid flow. The Hall current in this case has a marked effect on the magnitude and direction of the current density which will consequently affect the electromagnetic force term (Cramer & Pai, 1973).

In this chapter, we consider the unsteady MHD fluid flow between two parallel plates with heat transfer while putting into consideration the effect of the Hall current. Two cases are considered, first with the fluid flow between horizontal parallel plates and later a fluid flow between two vertical parallel plates.

3.1 Fluid flow between two horizontal parallel plates with one plate moving

Consider an incompressible, viscous, heat and electrically conducting fluid flowing between two infinite non conducting porous horizontal plates. The fluid flow is unsteady and a magnetic field is applied perpendicular to the plates. The upper plate is moving with a constant velocity U_0 while the lower plate is stationary. The induced magnetic field is neglected by assuming a very small magnetic Reynold's number. For this reason, the uniform magnetic field B_0 is considered as the total magnetic field acting on the fluid. We are going to put into consideration the effect of the Hall current on the fluid flow studied in this current chapter. The effects of suction and injection are also considered.

3.1.1 Mathematical formulation

Let the two non conducting horizontal plates be located on the planes $y = \pm h$ with the plates being of infinite length in the x and z directions i.e.$-\infty< x < \infty$ and $-\infty< z < \infty$. Further, consider the fluid to be flowing between the two plates under the influence of a constant pressure gradient $\frac{dP}{dx}$ in the x direction, and a uniform injection from below and suction from above with a constant velocity v_0. The pressure gradient is applied at time $t = 0$. The upper plate that is initially at rest is set into motion at time $t = 0$. A uniform magnetic field B_0 acts on the whole system in the positive y direction as shown in figure 2.1 on page 22.

For the fluid flow in consideration, all quantities depend on the space coordinate y and time t except the pressure gradient $\frac{dP}{dx}$ which is assumed constant. For an incompressible fluid the density of the fluid is assumed to be constant and in this case the continuity equation takes the form $\frac{\partial v}{\partial y} = 0$ which on integration gives $v=$ constant. This constant is equal to the suction velocity v_0. In particular, the velocity of the fluid is given as $\mathbf{q}(y,t) = u(y,t)\mathbf{i} + v_0\mathbf{j} + w(y,t)\mathbf{k}$. The applied magnetic field acts along the y axis and is given by $\boldsymbol{B}_0 = \langle 0, B_0, 0\rangle$.When the strength of the magnetic field is large, Ohm's law must be modified to include the Hall currents. If the ion slip and thermoelectric effects are neglected, we have Ohm's law given as (Cowling, 1957).

$$\mathbf{J} + \frac{\omega_e\tau_e}{H_0}(\mathbf{J}\times\mathbf{H}) = \sigma(\mathbf{E} + \mu_e\mathbf{q}\times\mathbf{H} + \frac{1}{e\eta_e}\bigtriangledown P_e) \tag{3.1}$$

Considering a short circuit problem, the applied electric field $\mathbf{E} = 0$ and for partially ionized gases the electron pressure gradient may be neglected. If $\mathbf{E} = 0$ and neglecting the pressure gradient, equation (3.1) takes the form

$$J_x\boldsymbol{i} + J_y\boldsymbol{j} + J_z\boldsymbol{k} + \frac{\omega_e\tau_e}{H_0}\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ J_x & J_y & J_z \\ 0 & H_0 & 0 \end{vmatrix} = \sigma\mu_e\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u & v_0 & w \\ 0 & H_0 & 0 \end{vmatrix}$$

$$\begin{pmatrix} J_x \\ J_y \\ J_z \end{pmatrix} + \frac{\omega_e\tau_e}{H_0}\begin{pmatrix} -H_0J_z \\ 0 \\ H_0J_x \end{pmatrix} = \sigma\mu_e\begin{pmatrix} -wH_0 \\ 0 \\ uH_0 \end{pmatrix}$$

$$J_x - \omega_e \tau_e J_z = -\sigma \mu_e H_0 w \tag{3.2}$$

$$J_z + \omega_e \tau_e J_x = \sigma \mu_e H_0 u \tag{3.3}$$

Here, the magnetic field intensity H_0 relates to the magnetic induction B_0 by the relation $\mathbf{B_0} = \mu_e \mathbf{H_0}$. The Hall parameter $m = \omega_e \tau_e$ and the equations (3.2) and (3.3) can be represented as

$$J_x - mJ_z = -\sigma \mu_e H_0 w \tag{3.4}$$

$$J_z + mJ_x = \sigma \mu_e H_0 u \tag{3.5}$$

or in matrix form

$$\begin{pmatrix} 1 & -m \\ m & 1 \end{pmatrix} \begin{pmatrix} J_x \\ J_z \end{pmatrix} = \begin{pmatrix} -\sigma \mu_e H_0 w \\ \sigma \mu_e H_0 u \end{pmatrix}$$

Solution of the equations (3.4) and (3.5) give the components of the electric current density along the x and z axes respectively as

$$J_x = \frac{\begin{vmatrix} -\sigma \mu_e H_0 w & -m \\ \sigma \mu_e H_0 u & 1 \end{vmatrix}}{1+m^2} = \frac{\sigma \mu_e H_0}{1+m^2}(mu - w) \tag{3.6}$$

$$J_z = \frac{\begin{vmatrix} 1 & -\sigma \mu_e H_0 w \\ m & \sigma \mu_e H_0 u \end{vmatrix}}{1+m^2} = \frac{\sigma \mu_e H_0}{1+m^2}(u + mw) \tag{3.7}$$

The electromagnetic force term is evaluated as

$$\boldsymbol{J} \times \mathbf{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ J_x & 0 & J_z \\ 0 & B_0 & 0 \end{vmatrix}$$

and the components of the electromagnetic force are given in equations (3.8) and (3.9) below.

$$(\mathbf{J} \times \mathbf{B})_x = -B_0 J_z = \frac{-\sigma \mu_e^2 H_0^2}{1+m^2}(u + mw) \tag{3.8}$$

$$(\mathbf{J} \times \mathbf{B})_z = B_0 J_x = \frac{\sigma \mu_e^2 H_0^2}{1+m^2}(mu - w) \tag{3.9}$$

Considering the velocity in the form $\mathbf{q}(y,t) = u(y,t)\mathbf{i} + v_0\mathbf{j} + w(y,t)\mathbf{k}$ the viscous term $\mu\nabla^2\boldsymbol{q}$ in the momentum conservation equation takes the form

$$\begin{aligned}\mu\nabla^2\boldsymbol{q}^* &= \mu\left(\frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial y^{*2}} + \frac{\partial^2}{\partial z^{*2}}\right)(u^*(y^*,t^*)\mathbf{i} + v_0^*\mathbf{j} + w^*(y^*,t^*)\mathbf{k}) \\ &= \mu\left(\boldsymbol{i}\frac{\partial^2 u^*}{\partial y^{*2}} + \boldsymbol{k}\frac{\partial^2 w^*}{\partial y^{*2}}\right) \end{aligned} \tag{3.10}$$

The (*) superscript has been used in equation (3.10) to represent the dimensional quantities. This is the notation that is used from this point onwards. Considering the viscous force term in equation (3.10) together with the electromagnetic force terms including the effect of Hall current in equations (3.8) and (3.9), we have the conservation of momentum equation written in component form as

$$\rho\left(\frac{\partial u^*}{\partial t^*} + v_0^*\frac{\partial u^*}{\partial y^*}\right) = -\frac{dP}{dx} + \mu\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u^* + mw^*)$$

and

$$\rho\left(\frac{\partial w^*}{\partial t^*} + v_0^*\frac{\partial w^*}{\partial y^*}\right) = \mu\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(mu^* - w^*)$$

or

$$\frac{\partial u^*}{\partial t^*} + v_0^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{dP}{dx} + \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(u^* + mw^*) \tag{3.11}$$

$$\frac{\partial w^*}{\partial t^*} + v_0^*\frac{\partial w^*}{\partial y^*} = \upsilon\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mu^* - w^*) \tag{3.12}$$

From equation (2.9) of chapter two, the viscous dissipation term is written as

$$\mu\phi^* = 2\mu\left[\left(\frac{\partial u^*}{\partial x^*}\right)^2 + \left(\frac{\partial v^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial z^*}\right)^2\right] + \left(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}\right)^2 + \left(\frac{\partial v^*}{\partial z^*} + \frac{\partial w^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial x^*} + \frac{\partial u^*}{\partial z^*}\right)^2$$

But the velocity for the flow in consideration is given as

$\mathbf{q}(y^*,t^*) = u^*(y^*,t^*)\ \mathbf{i} + v_0^*\boldsymbol{j} + w^*(y^*,t^*)\boldsymbol{k}$,

and therefore

$$\begin{aligned}\mu\phi &= 2(0) + \mu\left(\frac{\partial u^*(y^*,t^*)}{\partial y^*} + 0\right)^2 + \mu\left(0 + \frac{\partial w^*(y^*,t^*)}{\partial y^*}\right)^2 + (0)^2 \\ \mu\phi &= \mu\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right\}\end{aligned} \tag{3.13}$$

The Joule dissipation term is given as $\frac{J^{*2}}{\sigma}$. The electric current density from equations (3.6) and (3.7) is given as

$$\mathbf{J}^* = J_x^*\mathbf{i} + J_z^*\mathbf{k}$$

$$\mathbf{J}^* = \frac{\sigma\mu_e H_0}{1+m^2}(mu^* - w^*)\mathbf{i} + \frac{\sigma\mu_e H_0}{1+m^2}(u^* + mw^*)\mathbf{k} \tag{3.14}$$

Therefore $\mathrm{J}^{*2} = J^* \cdot J^*$ is evaluated as

$$\begin{aligned}
\mathbf{J}^* \cdot \mathbf{J}^* &= \frac{\sigma^2\mu_e^2 H_0^2}{(1+m^2)^2}\left[(mu^* - w^*)^2 + (u^* + mw^*)^2\right] \\
&= \frac{\sigma^2\mu_e^2 H_0^2}{(1+m^2)^2}\left[m^2(u^{*2} + w^{*2}) + u^{*2} + w^{*2}\right] \\
&= \frac{\sigma^2\mu_e^2 H_0^2}{1+m^2}(u^{*2} + w^{*2})
\end{aligned}$$

Therefore

$$J^{*2} = \frac{\sigma^2\mu_e^2 H_0^2}{1+m^2}(u^{*2} + w^{*2}) \tag{3.15}$$

and

$$\frac{J^{*2}}{\sigma} = \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u^{*2} + w^{*2}) \tag{3.16}$$

When the viscous effect and the Joule dissipation are considered, the energy conservation equation can be expressed as

$$\begin{aligned}
\rho C_p\left(\frac{\partial T^*}{\partial t^*} + v_0^*\frac{\partial T^*}{\partial y^*}\right) &= K\frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left\{\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right\} \\
&+ \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u^{*2} + w^{*2})
\end{aligned} \tag{3.17}$$

The concentration conservation equation is not affected by the Hall current and is given as

$$\frac{\partial C^*}{\partial t^*} + v_0^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{3.18}$$

For the fluid flow in consideration, let the two plates be initially isothermal at temperature T_1^*. The temperature of the upper plate is then raised to T_2^* and thereafter the lower and upper plates are maintained at two different but constant temperatures T_1^* and T_2^* respectively. Let the concentration at the plates be C_1^* initially. The concentration at the upper plate is then increased to C_2^* and the concentrations at the boundaries are kept constant thereafter. From the definition of the fluid flow problem under consideration, the initial and boundary conditions are respectively given as

$$\left.\begin{array}{l} u^*(-h^*,0) = w^*(-h^*,0) = 0 \\ u^*(h^*,0) = w^*(h^*,0) = 0 \\ T^*(-h^*,0) = T^*(h^*,0) = T_1^* \\ C^*(-h^*,0) = C^*(h^*,0) = C_1^* \end{array}\right\} \quad t^* = 0 \tag{3.19}$$

$$\left.\begin{array}{l} u^*(-h^*,t^*) = 0 \\ w^*(-h^*,t^*) = 0 \\ u^*(h^*,t^*) = U_0 \\ w^*(h^*,t^*) = 0 \\ T^*(-h^*,t^*) = T_1^* \\ T^*(h^*,t^*) = T_2^* \\ C^*(-h^*,t^*) = C_1^* \\ C^*(h^*,t^*) = C_2^* \end{array}\right\} \quad t^* > 0 \tag{3.20}$$

3.1.2 Non-dimensionalization

The equations governing the fluid flow are given by equations (3.11), (3.12), (3.17) and (3.18). If we consider the complex velocity vector in the form $\mathbf{q}^* = \mathbf{u}^* + i\mathbf{w}^*$, then the electromagnetic force terms in equations (3.8) and (3.9), per unit mass can be combined as follows.

$$\begin{aligned} \frac{\sigma\mu_e^2H_0^2}{\rho(1+m^2)}\{-(u^*+mw^*)+i(mu^*-w^*)\} &= \frac{\sigma\mu_e^2H_0^2}{\rho(1+m^2)}[m(-w^*+iu^*)-u^*-iw^*] \\ &= \frac{\sigma\mu_e^2H_0^2}{\rho(1+m^2)}(im\mathbf{q}^*-\mathbf{q}^*) \end{aligned} \tag{3.21}$$

$$\frac{\mathbf{J}^*\times\mathbf{B}^*}{\rho} = \frac{\sigma\mu_e^2H_0^2}{\rho(1+m^2)}\mathbf{q}^*(im-1) \tag{3.22}$$

From chapter two we had the momentum equation in the form

$$\rho\left(\frac{\partial\mathbf{q}^*}{\partial t^*}+v_0^*\frac{\partial\mathbf{q}^*}{\partial y^*}\right) = -\frac{dP^*}{dx^*}\boldsymbol{i}+\mu\frac{\partial^2\mathbf{q}^*}{\partial y^{*2}}-\sigma\mu_e^2H_o^2\mathbf{q}^*$$

But here the electromagnetic term is given by equation (3.22) and the momentum equation is written as

$$\rho\left(\frac{\partial\mathbf{q}^*}{\partial t^*}+v_0^*\frac{\partial\mathbf{q}^*}{\partial y^*}\right) = -\frac{dP^*}{dx^*}\boldsymbol{i}+\mu\frac{\partial^2\mathbf{q}^*}{\partial y^{*2}}-\frac{\sigma\mu_e^2H_0^2}{\rho(1+m^2)}\mathbf{q}^*(im-1)$$

Dividing each term by $\frac{U_0^3}{\upsilon}$ in the momentum equation we have

$$\rho \times \frac{\upsilon}{U_0^3}\left(\frac{\partial \mathbf{q}^*}{\partial t^*} + v_0^*\frac{\partial \mathbf{q}^*}{\partial y^*}\right) = -\frac{dP^*}{dx^*}\boldsymbol{i} \times \frac{\upsilon}{U_0^3} + \frac{\mu\upsilon}{U_0^3}\frac{\partial^2 \mathbf{q}^*}{\partial y^{*2}} - \frac{\upsilon}{U_0^3} \times \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}\mathbf{q}^*\,(im-1) \tag{3.23}$$

Using the scaling variables in equation (2.47) of chapter two and the non dimensional parameter $M^2 = \frac{\sigma\mu_e^2 H_0^2 \upsilon}{\rho U_0^2}$ given in chapter two, the non dimensional form of the last term in equation (3.23) becomes

$$\begin{aligned}
\frac{\upsilon}{U_0^3} \times \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}\mathbf{q}^*(im-1) &= \frac{\upsilon}{U_0^3} \times \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}U_0\mathbf{q}(im-1) \\
&= \frac{\upsilon}{U_0^3} \times \frac{\sigma\mu_e^2 H_0^2 U_0}{\rho}\mathbf{q}\frac{im-1}{1+m^2} \\
&= \frac{im-1}{1+m^2}M^2\mathbf{q}
\end{aligned} \tag{3.24}$$

Using the scaling variables in equation (2.47) of chapter two in the other terms of equation (3.23) the non dimensional form of the momentum conservation equation takes the form

$$\frac{\partial \mathbf{q}}{\partial t} + S\frac{\partial \mathbf{q}}{\partial y} = -\frac{dP}{dx}\boldsymbol{i} + \frac{\partial^2 \mathbf{q}}{\partial y^2} + M^2\frac{im-1}{1+m^2}\mathbf{q} \tag{3.25}$$

The equation (3.25) is written below in component form in the x and z axes directions.

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2}(u+mw) \tag{3.26}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + \frac{M^2}{1+m^2}(mu-w) \tag{3.27}$$

The viscous dissipation term is given as

$$\begin{aligned}
\mu\left(\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right) &= \frac{\mu U_0^4}{\upsilon^2}\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) \\
&= \frac{\mu U_0^4}{\upsilon^2}\left(\frac{\partial \mathbf{q}}{\partial y}\cdot\frac{\partial \bar{\mathbf{q}}}{\partial y}\right)
\end{aligned} \tag{3.28}$$

The Joule dissipation term of equation (3.16) can be written in non dimensional form as

$$\begin{aligned}\frac{\sigma\mu_e^2H_0^2}{1+m^2}(u^{*2}+w^{*2}) &= \frac{\sigma\mu_e^2H_0^2}{1+m^2}U_0^2(u^2+w^2)\\ &= \frac{\sigma\mu_e^2H_0^2U_0^2}{1+m^2}\mathbf{q}\bar{\mathbf{q}} \end{aligned} \tag{3.29}$$

Including the viscous dissipation term given in equation (3.28) and the Joule dissipation term given in equation (3.29) in the energy conservation equation (2.45) stated in chapter two, the energy conservation equation takes the form

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*}+v_0^*\frac{\partial T^*}{\partial y^*}\right)=K\frac{\partial^2T^*}{\partial y^{*2}}+\frac{\mu U_0^4}{\upsilon^2}\left(\frac{\partial \mathbf{q}}{\partial y}\cdot\frac{\partial \bar{\mathbf{q}}}{\partial y}\right)+\frac{\sigma\mu_e^2H_0^2U_0^2}{1+m^2}\mathbf{q}\bar{\mathbf{q}} \tag{3.30}$$

Multiplying the terms of equation (3.30) by $\frac{\upsilon}{U_0^2(T_2^*-T_1^*)}$ we have the viscous and Joule dissipation terms given as

$$\frac{\mu U_0^2}{\upsilon(T_2^*-T_1^*)}\left(\frac{\partial \mathbf{q}^*}{\partial y^*}\cdot\frac{\partial \bar{\mathbf{q}}^*}{\partial y^*}\right)=\rho C_pEc\frac{\partial \mathbf{q}}{\partial y}\cdot\frac{\partial \bar{\mathbf{q}}}{\partial y}$$

and

$$\frac{\sigma\mu_e^2H_0^2U_0^2\upsilon}{(1+m^2)\,U_0^2(T_2^*-T_1^*)}\mathbf{q}\bar{\mathbf{q}} = \frac{\rho C_pM^2Ec}{1+m^2}\mathbf{q}\bar{\mathbf{q}} \tag{3.32}$$

If this is done for all the terms of equation (3.30) and using the scaling variables and the non dimensional numbers of chapter two the energy conservation equation in non dimensional form can be written as

$$\frac{\partial\theta}{\partial t}+S\frac{\partial\theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\frac{\partial \boldsymbol{q}}{\partial y}\cdot\frac{\partial \bar{\boldsymbol{q}}}{\partial y}+\frac{EcM^2}{1+m^2}\mathbf{q}\bar{\mathbf{q}} \tag{3.33}$$

The mass conservation equation is of the form

$$\frac{\partial C}{\partial t}+S\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2C}{\partial y^2} \tag{3.34}$$

Using the scaling variables in equation (2.47) of chapter two, the non dimensional form of the initial conditions (3.19) can be written as follows

Since $u^*(y^*,t^*)=0$ and $w^*(y^*,t^*)=0$ then $u=\frac{u^*}{U_o}\Rightarrow u=\frac{0}{U_o}$ *or* $u=0$ and $w=\frac{w^*}{U_o}\Rightarrow w=\frac{0}{U_o}$ or w = 0

From **q** = u + iw we have **q** = 0

From the initial conditions $T^*(y^*, t^*) = T_1^*$ and $C^*(y^*, t^*) = C_1^*$ and the non dimensional numbers $\theta = \frac{T^*-T_1^*}{T_2^*-T_1^*}$ *and* $C = \frac{C^*-C_1^*}{C_2^*-C_1^*}$ we have $\theta = \frac{T_1^*-T_1^*}{T_2^*-T_1^*}$ or $\theta = 0$ *and* C $= \frac{C_1^*-C_1^*}{C_2^*-C_1^*}$ or C $= 0$

The initial conditions then take the form

$$\left.\begin{array}{l} q(-h,0) = q(h,0) = 0 \\ \theta(-h,0) = \theta(h,0) = 0 \\ C(-h,0) = C(h,0) = 0 \end{array}\right\} \qquad t = 0 \tag{3.35}$$

The boundary conditions can be similarly derived to give

$$\left.\begin{array}{l} q(-h,t) = 0 \\ q(h,t) = 1 \\ \theta(-h,t) = 0 \\ \theta(h,t) = 1 \\ C(-h,t) = 0 \\ C(h,t) = 1 \end{array}\right\} \qquad t > 0 \tag{3.36}$$

3.1.3 Solution

We seek a solution of the system of equations (3.25), (3.33) and (3.34) together with the initial conditions (3.35) and the boundary conditions (3.36). The system of equations is non linear and we apply the numerical approximation method of finite differences in the solution as described in section 2.3.2 on page 32. We use the forward differences as approximations to the derivatives. The finite difference form of the momentum conservation equations (3.26)and (3.27), the energy conservation equation (3.33) and the concentration equation (3.34) which govern the fluid flow is given as

$$\begin{aligned} \frac{u\,(i,j+1) - u(i,j)}{\Delta t} &= -\frac{dP}{dx} + \frac{u(i-1,j) - 2u(i,j) + u(i+1,j)}{(\Delta y)^2} \\ &- S\left\{\frac{u(i+1,j) - u(i,j)}{\Delta y}\right\} \\ &- \frac{M^2}{1+m^2}\left[u(i,j) + mw(i,j)\right] \end{aligned} \tag{3.37}$$

$$\begin{aligned}\frac{w\left(i,j+1\right)-w(i,j)}{\Delta t} &= \frac{w(i-1,j)-2w(i,j)+w(i+1,j)}{\left(\Delta y\right)^{2}} \\ &- S\left\{\frac{w(i+1,j)-w(i,j)}{\Delta y}\right\} \\ &+ \frac{M^{2}}{1+m^{2}}\left[mu(i,j)-w(i,j)\right] \qquad (3.38)\end{aligned}$$

$$\begin{aligned}\frac{\theta\left(i,j+1\right)-\theta(i,j)}{\Delta t} &= \frac{1}{Pr}\left[\frac{\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)}{(\Delta y)^{2}}\right] \\ &+ Ec\left[\left(\frac{u(i+1,j)-u(i,j)}{\Delta y}\right)^{2}+\left(\frac{w(i+1,j)-w(i,j)}{\Delta y}\right)^{2}\right] \\ &+ \frac{EcM^{2}}{1+m^{2}}\left[u(i,j)^{2}+w(i,j)^{2}\right]-S\left\{\frac{\theta(i+1,j)-\theta(i,j)}{\Delta y}\right\}\end{aligned}$$
(3.39)

$$\begin{aligned}\frac{C(i,j+1)-C(i,j)}{\Delta t} &= \frac{1}{Sc}\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta y)^{2}}\right] \\ &- S\left[\frac{C(i+1,j)-C(i,j)}{\Delta y}\right] \qquad (3.40)\end{aligned}$$

The finite difference form of the initial conditions (3.35),and the boundary conditions (3.36) is given below.

$$\left.\begin{array}{l} q(0,0)=q(40,0)=0 \\ \theta(0,0)=\theta(40,0)=0 \\ C(0,0)=C(40,0)=0 \end{array}\right\} j=0 \qquad (3.41)$$

$$\left.\begin{array}{l} q(0,j)=0 \\ q(40,j)=1 \\ \theta(0,j)=0 \\ \theta(40,j)=1 \\ C(0,j)=0 \\ C(40,j)=1 \end{array}\right\} j>0 \qquad (3.42)$$

In this case i and j represent y and t respectively. Rearranging each of these equations enables us to compute consecutive terms of the velocities u and w, the temperature θ and the concentration C using the initial values and boundary conditions given in equations (3.41) and (3.42). Rearrangement of the equations (3.37) to (3.40) gives

$$\begin{aligned} u(i,j+1) &= -\Delta t \frac{dP}{dx} + \frac{\Delta t}{(\Delta y)^2}\left[u(i-1,j) - 2u(i,j) + u(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta y}\left[u(i+1,j) - u(i,j)\right] - \left[\frac{\Delta t M^2}{1+m^2} - 1\right]u(i,j) \\ &- \Delta t\left[\frac{mM^2}{1+m^2}\right]w(i,j) \end{aligned} \tag{3.43}$$

$$\begin{aligned} w(i,j+1) &= \frac{\Delta t}{(\Delta y)^2}\left[w(i-1,j) - 2w(i,j) + w(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta y}\left[w(i+1,j) - w(i,j)\right] \\ &- \left[\frac{\Delta t M^2}{1+m^2} - 1\right]w(i,j) + \Delta t\left[\frac{mM^2}{1+m^2}\right]u(i,j) \end{aligned} \tag{3.44}$$

$$\begin{aligned} \theta(i,j+1) &= \frac{\Delta t}{\Pr(\Delta y)^2}\left[\theta(i-1,j) - 2\theta(i,j) + \theta(i+1,j)\right] - S\frac{\Delta t}{\Delta y}\theta(i+1,j) \\ &+ Ec\frac{\Delta t}{(\Delta y)^2}\left[\{u(i+1,j) - u(i,j\}^2 + \{w(i+1,j) - w(i,j\}^2\right] \\ &+ Ec\frac{M^2\Delta t}{1+m^2}\left[u(i,j)^2 + w(i,j)^2\right] + \left[S\frac{\Delta t}{\Delta y} + 1\right]\theta(i,j) \end{aligned} \tag{3.45}$$

$$\begin{aligned} C(i,j+1) &= \frac{\Delta t}{Sc(\Delta y)^2}\left[C(i-1,j) - 2C(i,j) + C(i+1,j)\right] - \frac{S\Delta t}{\Delta y}C(i+1,j) \\ &+ \left\{\frac{S\Delta t}{\Delta y} + 1\right\}C(i,j) \end{aligned} \tag{3.46}$$

3.1.4 Observations and discussions

Computations for the primary velocity u, the secondary velocity w, the temperature θ and the concentration C were made for $\Pr = 0.71$ corresponding to air and $M^2 = 6.0$ representing a strong magnetic field. The parameters that were varied included the pressure gradient $\frac{dP}{dx}$, the Hall parameter m, the suction parameter

S, the Schmidt number Sc and the Ekcert number Ec. The reference values represented in figures 3.1 on page 48 to figure 3.4 on page 51 by the curve labelled "test" are $\frac{dP}{dx} = 5.0$, m = 1.0, S = -0.5, Sc = 0.4 and $Ec = 0.2$. These values of the parameters were varied one at a time and input into a C++ computer program. Computations were done using equations (3.43) to (3.46), the initial conditions (3.41) and the boundary conditions (3.42) and curves plotted for each case. These results were then represented in the figures labelled figure 3.1 on page 48 to figure 3.4 on page 51.

(a) Primary velocity profiles

We discuss how each of the parameters affects the primary velocity profiles (u) of the fluid flow as represented by the graphs in figure 3.1 on page 48.

Pressure gradient

When the pressure gradient is positive it leads to an increase in the primary velocity profiles in the negative direction as observed for the curves in figure 3.1. When the pressure gradient is negative then this pressure gradient leads to an increase in the primary velocity profiles in the positive direction as observed in one of the curves $\left(\frac{dP}{dx} = -5.0\right)$ of figure 3.1. It is observed that except for $\frac{dP}{dx} < 0$, the primary velocity increases in the opposite direction at the stationary boundary. Away from the boundaries, the velocity is maintained at a constant level. As the moving boundary is approached, the velocity then reduces up to a point when u = 0 (stationary point) and then it increases suddenly until it reaches the velocity of the moving plate. This can be supported by the fact that the pressure gradient force term $\frac{F_x}{m} = -\frac{1}{\rho}\frac{dP}{dx}$ along the x axis is proportional to the acceleration $\frac{dv_x}{dt}$. When $\frac{dP}{dx}$ is positive, then the pressure force term acts in the opposite direction to the direction of the fluid flow. On the other hand, when $\frac{dP}{dx}$ is negative, then the pressure force term acts in the same direction as that of the fluid flow hence aiding the fluid flow.

Suction

When there is no suction (S = 0), the magnitude of the fluid velocity increases near both the stationary and moving plates. This means that the effect of introducing suction retards the fluid flow due to the convection of the fluid across the plates.

Hall parameter

As the value of the Hall parameter m increases (m = 2.0), figure 3.1 on the following page shows that the free stream velocity is achieved further away from the plates and the free stream region is narrower. The magnitude of the primary velocity profiles increases near both plates. This can be attributed to the effect of an increase in the cyclotron frequency and thus the rotation of the electrons and/or an increase in the collision times of the electrons with an increase in the value of the Hall parameter. The Hall parameter is directly proportional to the cyclotron frequency and the electron collision times and an increase in the value of the Hall parameter would be due to an increase in either or both of these. This agrees with the observations made by Attia in (Attia, 2006).

(b) Secondary velocity profiles

We discuss how each of the parameters affects the secondary velocity profiles (w) of the fluid flow as seen in figure 3.2 on the next page. We observe a general trend of the magnitude of the velocity increasing near the lower plate to a maximum and then maintaining a constant value before either increasing in the case of $\frac{dP}{dx} < 0$ or decreasing for $\frac{dP}{dx} > 0$.

Pressure gradient

A positive pressure gradient leads to an increase in the secondary velocity profiles in the negative direction while a negative pressure gradient leads to an increase in the primary velocity profiles in the positive direction the secondary velocity near the lower stationary plate as observed in figure 3.2. Near the stationary lower plate, the magnitude of the velocity increases to a maximum before stagnating at a constant level. The increase is positive for a negative pressure gradient and negative for a positive pressure gradient. As you approach the moving plate for $\frac{dP}{dx} < 0(= -5.0)$, the velocity increases further to a maximum before reducing

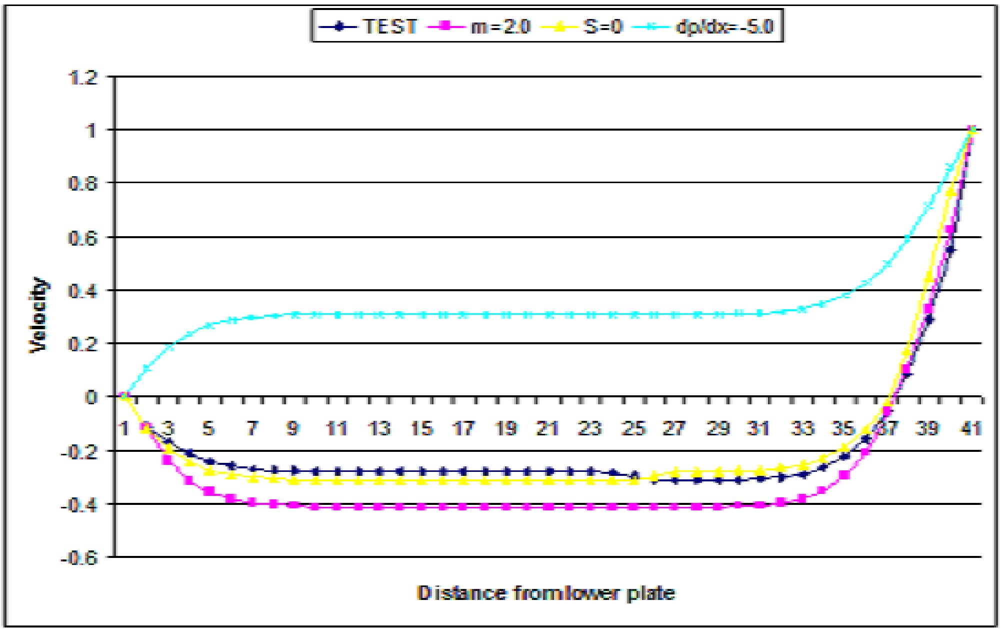

FIGURE 3.1. Primary velocity profiles for horizontal plates with one plate moving

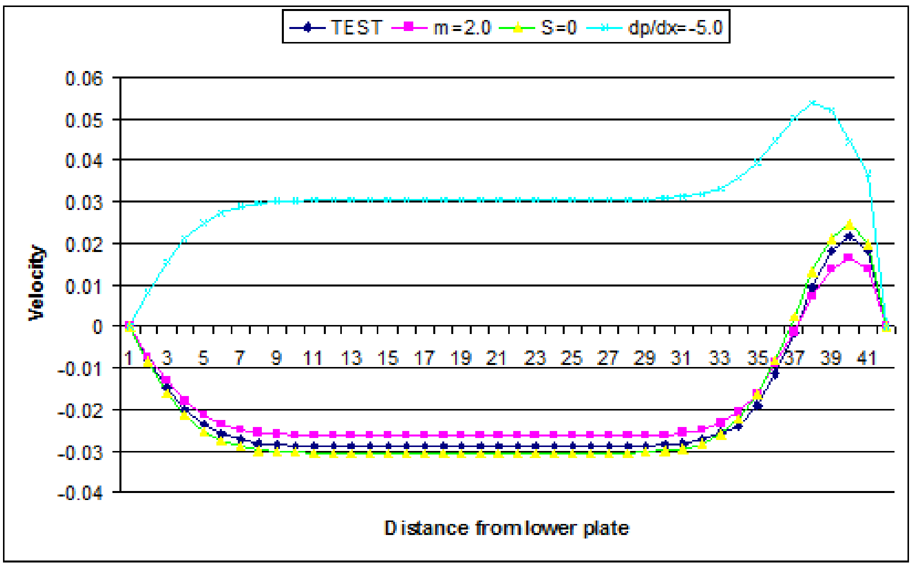

FIGURE 3.2. Secondary velocity profiles for horizontal plates with one plate moving

to zero at the moving plate. On the other hand, as you approach the moving plate for $\frac{dP}{dx} > 0 (= 5.0)$ the speed reduces to zero then increases to a maximum before again reducing to zero at the moving plate.

Suction

Absence of suction (S = 0) leads to an increase in the velocity profiles near the stationary plate. As the moving plate is approached, the velocity reduces at a faster rate. Stagnation velocity is reached further away from the moving plate and then the velocity increase is more up to a maximum adjacent to the moving plate before reducing to zero at the plate. This means that the effect of introducing suction retards the fluid flow which can be attributed to the convection of the fluid across the plates.

Hall parameter

An increase in the Hall parameter (m = 2.0) leads to a decrease in the secondary velocity near the stationary plate. The region for a constant velocity for a larger value of the Hall parameter is narrower. As the moving plate is approached, the velocity then increases at a faster rate to a lesser maximum before reducing to zero at the plate.

(c) Temperature profiles

A change in each of the parameters has no significant effect near the stationary plate but the effects are observed as you get closer to the moving plate. Considering figure 3.3 on page 51, we discuss the effect of each of the parameters below.

Pressure gradient

When the imposed pressure gradient is negative, there is a decrease of temperature near the moving plate.

Hall parameter

An increase in the Hall parameter (m = 2.0) leads to a slight decrease in the temperature profiles. This can be attributed to the effect of an increase in the Hall parameter causing a decrease in the Joule dissipation which is proportional to $\frac{1}{1+m^2}$.

Suction

Absence of suction (S = 0) leads to the temperature increasing near the moving plate. This is because the introduction of suction contributes to convection which pumps the fluid towards the upper plate, a result that agrees with Attia in (Attia, 2006).

Eckert number

A reduction in the value of the Eckert number (Ec = 0.02) leads to a decrease in the temperature near the moving plate. This is because an increase in the Eckert number leads to a decrease in the thermal energy and consequently a decrease in the temperature profiles.

(d) Concentration profiles

From figure 3.4 on the next page it is observed that there is little effect produced by the change in the parameters near the stationary plate but the effects are observed as you approach the moving plate.

Schmidt number

An increase in the value of the Schmidt number (Sc = 0.7) causes a decrease in the concentration closer to the moving plate. The mass diffusion parameter Sc is inversely proportional to the concentration and therefore its increase results in a decrease in the concentration profiles.

Suction

Removal of suction (S = 0) leads to the concentration profiles increasing near the moving plate. This can be attributed to removal of the convection of the fluid across the plates.

3.2 Fluid flow between two vertical parallel plates with one plate moving

The fluid flow discussed in this section is unsteady, laminar and fully developed. The fluid is incompressible, viscous, heat and electrically conducting and flows between two infinite non conducting porous vertical plates. The theory discussed in chapter two is extended here whereby the effect of the Hall current is put into consideration when analyzing the fluid flow.

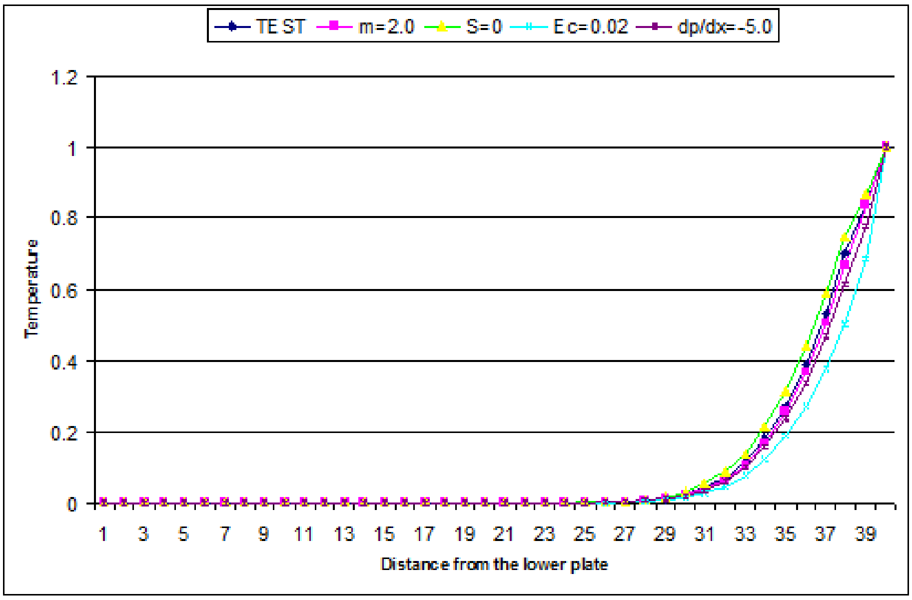

FIGURE 3.3. Temperature profiles for horizontal plates with one plate moving

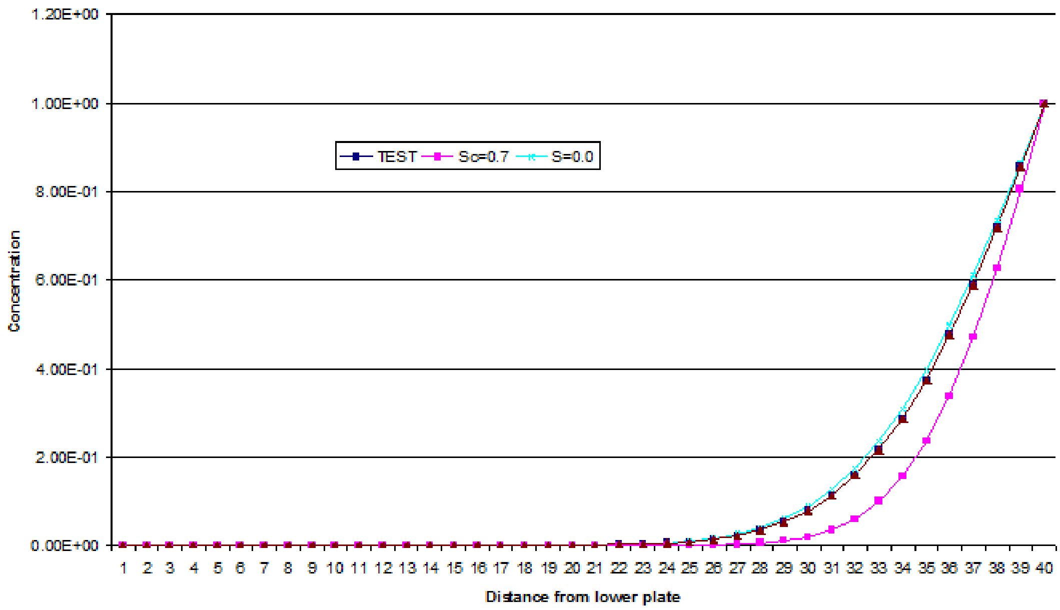

FIGURE 3.4. Concentration profiles for horizontal plates with one plate moving

3.2.1 Mathematical formulation

Consider an unsteady, laminar, hydromagnetic fully developed fluid flowing between two parallel vertical plates. The physical configuration is similar to that described in figure 2.2 on page 28. The parallel vertical walls are located on the planes $x^* = -L$ and $x^* = L$ and extend infinitely in the y^* and z^* axes. A constant magnetic field of strength B_0^* is applied across the parallel plates in the positive x^* axis direction. A second material is injected uniformly from the left and there is uniform suction from the right with velocity u_0 applied at time $t^* = 0$. The equation of continuity is given as $\frac{\partial u^*}{\partial x^*} = 0$ which on integration gives $u=$ constant with this constant being the constant suction velocity.

$$u^* = u_0^* \tag{3.47}$$

Existence of the Hall term gives rise to a z^* component of the velocity and in this case the velocity vector of the fluid is given by

$$\boldsymbol{q}^*(x^*, t^*) = u_0^* \boldsymbol{i} + v^*(x^*, t^*) \boldsymbol{j} + w^*(x^*, t^*) \boldsymbol{k} \tag{3.48}$$

Just as was discussed in the previous section 3.1.1, Ohm's law with Hall current effect included is given in equation (3.1). For the case in consideration, we have from equation (3.1),

$$\begin{pmatrix} J_x^* \\ J_y^* \\ J_z^* \end{pmatrix} + \frac{m}{H_0} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ J_x^* & J_y^* & J_z^* \\ H_0 & 0 & 0 \end{vmatrix} = \sigma \mu_e \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_0^* & v^* & w^* \\ H_0 & 0 & 0 \end{vmatrix}$$

$$\begin{pmatrix} J_x^* \\ J_y^* \\ J_z^* \end{pmatrix} + \frac{m}{H_0} \begin{pmatrix} 0 \\ H_0 J_z^* \\ -H_0 J_y^* \end{pmatrix} = \sigma \mu_e H_0 \begin{pmatrix} 0 \\ w^* \\ -v^* \end{pmatrix}$$

In component form we have

$$\left.\begin{array}{l} J_x^* = 0 \\ J_y^* + m J_z^* = \sigma \mu_e H_0 w^* \\ J_z^* - m J_y^* = -\sigma \mu_e H_0 v^* \end{array}\right\} \tag{3.49}$$

On solving the system of equations (3.49) simultaneously we have the components of the electric current density along the y^* and z^* axes given respectively as

$$J_y^* = \frac{\begin{vmatrix} \sigma\mu_e H_0 w^* & m \\ -\sigma\mu_e H_0 v^* & 1 \end{vmatrix}}{\begin{vmatrix} 1 & m \\ -m & 1 \end{vmatrix}} = \frac{\sigma\mu_e H_0}{1+m^2}(w^* + mv^*)$$

$$J_z^* = \frac{\begin{vmatrix} 1 & \sigma\mu_e H_0 w^* \\ -m & -\sigma\mu_e H_0 v^* \end{vmatrix}}{1+m^2} = \frac{\sigma\mu_e H_0}{1+m^2}(mw^* - v^*)$$

i.e.

$$\begin{aligned} J_y^* &= \tfrac{\sigma\mu_e H_0}{1+m^2}(mv^* + w^*) \\ J_z^* &= \tfrac{\sigma\mu_e H_0}{1+m^2}(mw^* - v^*) \end{aligned} \tag{3.50}$$

The electromagnetic force term per unit mass is given by

$$\frac{\mathbf{J}^* \times \mathbf{B}^*}{\rho} = \frac{1}{\rho}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & J_y^* & J_z^* \\ B_0 & 0 & 0 \end{vmatrix} = \frac{B_0}{\rho}\begin{pmatrix} 0 \\ J_z^* \\ -J_y^* \end{pmatrix} \tag{3.51}$$

The component form of equation (3.51) is

$$\left(\frac{\mathbf{J}^* \times \mathbf{B}^*}{\rho}\right)_y = \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mw^* - v^*) \tag{3.52}$$

$$\left(\frac{\mathbf{J}^* \times \mathbf{B}^*}{\rho}\right)_z = -\frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mv^* + w^*) \tag{3.53}$$

A pressure gradient will exist in the y^* direction as a result of the change in elevation. By considering the Boussinesq approximation, and taking the density to depend on temperature and concentration, the body force term takes the form of equation (2.62) in chapter two. The equation of conservation of momentum with the electromagnetic force terms in equations (3.52) and (3.53) therefore takes the form

$$\frac{\partial v^*}{\partial t^*} + u_0^*\frac{\partial v^*}{\partial x^*} = \upsilon\frac{\partial^2 v^*}{\partial x^{*2}} + g\left[\beta\left(T^* - T_1^*\right) + \beta_c\left(C^* - C_1^*\right)\right] + \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mw^* - v^*) \tag{3.54}$$

and

$$\frac{\partial w^*}{\partial t^*} + u_0^*\frac{\partial w^*}{\partial x^*} = \upsilon\frac{\partial^2 w^*}{\partial x^{*2}} - \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mv^* + w^*) \tag{3.55}$$

The viscous term in the momentum conservation equation has been taken in the form

$$\mu\nabla^2\mathbf{q}^* = \mu\left(\frac{\partial^2}{\partial x^{*2}}+\frac{\partial^2}{\partial y^{*2}}+\frac{\partial^2}{\partial z^{*2}}\right)(u_0^*\mathbf{i}+v^*(x^*,t^*)\mathbf{j}+w^*(x^*,t^*)\mathbf{k})$$

$$= \mu\left(\mathbf{j}\frac{\partial^2 v^*}{\partial x^{*2}}+\mathbf{k}\frac{\partial^2 w^*}{\partial x^{*2}}\right) \tag{3.56}$$

We consider the viscous dissipation term as

$$\mu\left(\left(\frac{\partial v^*}{\partial x^*}\right)^2+\left(\frac{\partial w^*}{\partial x^*}\right)^2\right) \tag{3.57}$$

The Joule dissipation term is taken as $\frac{J^{*2}}{\sigma}$. The electric current density from equations (3.50) is given as

$$\mathbf{J}^* = J_y^*\mathbf{j}+J_z^*\mathbf{k} = \frac{\sigma\mu_e H_0}{1+m^2}(mv^*+w^*)\mathbf{j}+\frac{\sigma\mu_e H_0}{1+m^2}(mw^*-v^*)\mathbf{k} \tag{3.58}$$

$$J^{*2} = \mathbf{J}^*\cdot\mathbf{J}^* = \frac{\sigma^2\mu_e^2H_0^2}{(1+m^2)^2}\left[(mv^*+w^*)^2+(mw^*-v^*)^2\right]$$

$$J^{*2} = \frac{\sigma^2\mu_e^2H_0^2}{1+m^2}(v^{*2}+w^{*2}) \tag{3.59}$$

The Joule dissipation term is then given as

$$\frac{J^{*2}}{\sigma} = \frac{\sigma\mu_e^2H_0^2}{1+m^2}(v^{*2}+w^{*2}) \tag{3.60}$$

When the viscous term from equation (3.57) and Joule dissipation term from equation (3.60) are considered, the energy conservation is given by the equation

$$\frac{\partial T^*}{\partial t^*}+u_0^*\frac{\partial T^*}{\partial x^*} = \frac{K}{\rho C_p}\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\upsilon}{C_p}\left(\left(\frac{\partial v^*}{\partial x^*}\right)^2+\left(\frac{\partial w^*}{\partial x^*}\right)^2\right)+\frac{\sigma\mu_e^2H_0^2}{\rho C_p(1+m^2)}(v^{*2}+w^{*2}) \tag{3.61}$$

The equation of concentration conservation is given as

$$\frac{\partial C^*}{\partial t^*}+u_0^*\frac{\partial C^*}{\partial x^*} = D\frac{\partial^2 C^*}{\partial x^{*2}} \tag{3.62}$$

The two plates are initially stationary but at time $t^*=0$ the plate at $x^*=L^*$ is set into motion with a velocity U_0 along its plane. The two plates are considered to be initially isothermal at a temperature T_1^*. At the time t = 0, the temperature of the plate at the boundary $x^* = L^*$ is raised to T_2^* and then kept constant thereafter with $T_2^* > T_1^*$. The concentration of the fluid at the plates is initially taken as C_1^*. However, at time t = 0, the concentration at the boundary $x^* = L^*$ is increased to C_2^*. Thereafter, the concentrations are maintained at C_1^* and C_2^* at the boundaries $x^* = -L^*$ and $x^* = L^*$ respectively. Considering the no slip condition, the initial conditions for this fluid flow configuration are given as

$$\left.\begin{array}{l} v^*(-L^*,0) = w^*(-L^*,0) = 0 \\ v^*(L^*,0) = w^*(L^*,0) = 0 \\ T^*(-L^*,0) = T^*(L^*,0) = T_1^* \\ C^*(-L^*,0) = C^*(L^*,0) = C_1^* \end{array}\right\} \quad t^* = 0 \tag{3.63}$$

The boundary conditions as described above for this fluid flow in consideration are given as

$$\left.\begin{array}{l} v^*(-L^*,t^*) = 0 \\ w^*(-L^*,t^*) = 0 \\ v^*(L^*,t^*) = U_0 \\ w^*(L^*,t^*) = 0 \\ T^*(-L^*,t^*) = T_1^* \\ T^*(L^*,t^*) = T_2^* \\ C^*(-L^*,t^*) = C_1^* \\ C^*(L^*,t^*) = C_2^* \end{array}\right\} \quad t^* > 0 \tag{3.64}$$

3.2.2 Non-dimensionalization

To get a more general solution to the system of equations, we non dimensionalise the equations governing the fluid flow given by the equations of momentum conservation (3.54) and (3.55), equation of conservation of energy (3.61) and equation of concentration conservation (3.62). In this section we consider the complex velocity vector in the form $q^* = v^* + iw^*$. Considering the scaling variables in equation (2.47) of chapter two, the non dimensional parameters of chapter two and letting $v = \frac{v^*}{U_0}$ and $w = \frac{w^*}{U_0}$ we can proceed with the non dimensionalization process. The body force term of the momentum conservation equation (3.54) is non- dimensionalised to become

$$g\left[\beta\left(T^* - T_1^*\right) + \beta_c\left(C^* - C_1^*\right)\right] \div \frac{U_0^3}{\upsilon} = Gr\theta + GcC \tag{3.65}$$

where Gc is the modified Grashof number given as

$$Gc = \frac{\upsilon g\beta_c \left(C_2^* - C_1^*\right)}{U_0^3}$$

The electromagnetic force terms in equations (3.52) and (3.53) become

$$\begin{aligned} \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mw^* - v^*) \div \frac{U_0^3}{\upsilon} &= \frac{\sigma\mu_e^2 H_0^2 \upsilon}{\rho(1+m^2)U_0^2}\left[mw - v\right] \\ &= \frac{M^2}{(1+m^2)}\left[mw - v\right] \end{aligned} \tag{3.66}$$

$$\begin{aligned} -\frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mv^* + w^*) \div \frac{U_0^3}{\upsilon} &= -\frac{\sigma\mu_e^2 H_0^2 \upsilon}{\rho(1+m^2)U_0^2}\left[mv + w\right] \\ &= -\frac{M^2}{(1+m^2)}\left[mv + w\right] \end{aligned} \tag{3.67}$$

The non dimensional form of the momentum conservation equations in this case along the y and z axes are given respectively as

$$\frac{\partial v}{\partial t} + S\frac{\partial v}{\partial x} = \frac{\partial^2 v}{\partial x^2} + Gr\theta + GcC + \frac{M^2}{(1+m^2)}\left[mw - v\right] \tag{3.68}$$

and

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial x} = \frac{\partial^2 w}{\partial x^2} - \frac{M^2}{(1+m^2)}\left[mv + w\right] \tag{3.69}$$

In the energy conservation equation, the viscous dissipation term is non dimensionalized as follows

$$\begin{aligned} \frac{\mu}{\rho C_p}\left(\left(\frac{\partial v^*}{\partial x^*}\right)^2 + \left(\frac{\partial w^*}{\partial x^*}\right)^2\right) \times \frac{\upsilon}{U_0^2(T_2^* - T_1^*)} &= \frac{U_0^2}{C_p(T_2^* - T_1^*)}\left(\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right) \\ &= Ec\left(\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right) \end{aligned} \tag{3.70}$$

The Joule dissipation term is given as

$$\begin{aligned} \frac{\sigma\mu_e^2 H_0^2}{\rho C_p\left(1+m^2\right)}(v^{*2} + w^{*2}) \times \frac{\upsilon}{U_0^2(T_2^* - T_1^*)} &= \frac{\sigma\mu_e^2 H_0^2 U_0^2 \upsilon}{\rho C_p U_0^2(T_2^* - T_1^*)\left(1+m^2\right)}(v^2 + w^2) \\ &= \frac{M^2 Ec}{1+m^2}\left(v^2 + w^2\right) \end{aligned} \tag{3.71}$$

The energy equation (3.61) can therefore be written in non dimensional form as

$$\frac{\partial \theta}{\partial t} + S\frac{\partial \theta}{\partial x} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial x^2} + E_C\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] + \frac{M^2 Ec}{1+m^2}(v^2 + w^2) \tag{3.72}$$

The mass conservation equation in non dimensional form is

$$\frac{\partial C}{\partial t} + S\frac{\partial C}{\partial x} = \frac{1}{Sc}\frac{\partial^2 C}{\partial x^2} \tag{3.73}$$

The non dimensional form of the initial and boundary conditions (3.63) and (3.64) are respectively given as

$$\left.\begin{array}{l} v(-L,0) = w(-L,0) = 0 \\ v(L,0) = w(L,0) = 0 \\ \theta(-L,0) = \theta(L,0) = 0 \\ C(-L,0) = C(L,0) = 0 \end{array}\right\} \quad t = 0 \tag{3.74}$$

$$\left.\begin{array}{l} v(-L,t) = 0 \\ w(-L,t) = 0 \\ v(L,t) = 1 \\ w(L,t) = 0 \\ \theta(-L,t) = 0 \\ \theta(L,t) = 1 \\ C(-L,t) = 0 \\ C(L,t) = 1 \end{array}\right\} \quad t > 0 \tag{3.75}$$

3.2.3 Solution

The solution of the velocities, temperature and concentration is sought from the system of equations (3.68), (3.69), (3.72) and (3.73) together with the initial conditions (3.74) and the boundary conditions (3.75). Since the system of equations is non linear, we apply the numerical approximation method of finite differences to get the solution to this system of equations. Replacing the derivatives in these equations by finite differences and letting i and j represent x and t respectively at the mesh points we have $x \pm \Delta x = i \pm 1$, $t \pm \Delta t = j \pm 1$ and the finite difference equations take the form given below.

$$\begin{aligned} \frac{v\,(i,j+1) - v(i,j)}{\Delta t} &= \frac{v(i-1,j) - 2v(i,j) + v(i+1,j)}{(\Delta x)^2} \\ &+ Gr\theta(i,j) + GcC(i,j) + \frac{M^2}{1+m^2}\left[mw(i,j) - v(i,j)\right] \\ &- S\left\{\frac{v(i+1,j) - v(i,j)}{\Delta x}\right\} \end{aligned} \tag{3.76}$$

$$\begin{aligned}
\frac{w\,(i,j+1)-w(i,j)}{\Delta t} &= \frac{w(i-1,j)-2w(i,j)+w(i+1,j)}{(\Delta x)^2} \\
&- S\left\{\frac{w(i+1,j)-w(i,j)}{\Delta x}\right\} \\
&- \frac{M^2}{1+m^2}\left[mv(i,j)+w(i,j)\right]
\end{aligned} \tag{3.77}$$

$$\begin{aligned}
\frac{\theta\,(i,j+1)-\theta(i,j)}{\Delta t} &= \frac{1}{Pr}\left[\frac{\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)}{(\Delta x)^2}\right] \\
&- S\left\{\frac{\theta(i+1,j)-\theta(i,j)}{\Delta x}\right\} \\
&+ Ec\left[\left(\frac{v(i+1,j)-v(i,j)}{\Delta x}\right)^2+\left(\frac{w(i+1,j)-w(i,j)}{\Delta x}\right)^2\right] \\
&+ \frac{EcM^2}{1+m^2}\left[v^2(i,j)+w^2(i,j)\right]
\end{aligned} \tag{3.78}$$

$$\begin{aligned}
\frac{C(i,j+1)-C(i,j)}{\Delta t} &= \frac{1}{Sc}\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta x)^2}\right] \\
&- S\left[\frac{C(i+1,j)-C(i,j)}{\Delta x}\right]
\end{aligned} \tag{3.79}$$

The finite difference form of the initial conditions (3.74),and the boundary conditions (3.75) is given below.

$$\left.\begin{array}{l}
q(-40,0)=q(40,0)=0 \\
\theta(-40,0)=\theta(40,0)=0 \\
C(-40,0)=C(40,0)=0
\end{array}\right\} \quad j=0 \tag{3.80}$$

$$\left.\begin{array}{l}
q(-40,j)=0 \\
q(40,j)=1 \\
\theta(-40,j)=0 \\
\theta(40,j)=1 \\
C(-40,j)=0 \\
C(40,j)=1
\end{array}\right\} \quad j>0 \tag{3.81}$$

Rearranging the terms of equations (3.76) to (3.79) we can compute consecutive values of the velocities v and w, the temperature θ and the concentration C as shown in equations (3.82) to (3.85).

$$\begin{aligned}
v(i,j+1) &= \frac{\Delta t}{(\Delta x)^2}\left[v(i-1,j)-2v(i,j)+v(i+1,j)\right] \\
&- \frac{S\Delta t}{\Delta x}\left[v(i+1,j)-v(i,j)\right]+\Delta t\left[Gr\theta(i,j)+GcC(i,j)\right] \\
&- \left[\frac{\Delta t M^2}{1+m^2}-1\right]v(i,j)\ +\Delta t\left[\frac{mM^2}{1+m^2}\right]w(i,j) \qquad (3.82)
\end{aligned}$$

$$\begin{aligned}
w(i,j+1) &= \frac{\Delta t}{(\Delta x)^2}\left[w(i-1,j)-2w(i,j)+w(i+1,j)\right] \\
&- \frac{S\Delta t}{\Delta x}\left[w(i+1,j)-w(i,j)\right] \\
&- \left[\frac{\Delta t M^2}{1+m^2}-1\right]w(i,j)-\Delta t\left[\frac{mM^2}{1+m^2}\right]v(i,j) \qquad (3.83)
\end{aligned}$$

$$\begin{aligned}
\theta(i,j+1) &= \frac{\Delta t}{\Pr(\Delta x)^2}\left[\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)\right] \\
&- S\frac{\Delta t}{\Delta x}\theta(i+1,j)+\left[S\frac{\Delta t}{\Delta x}+1\right]\theta(i,j) \\
&+ Ec\frac{\Delta t}{(\Delta x)^2}\left[\{v(i+1,j)-v(i,j\}^2+\{w(i+1,j)-w(i,j\}^2\right] \\
&+ Ec\frac{M^2\Delta t}{1+m^2}\left[v^2(i,j)+w^2(i,j)\right] \qquad (3.84)
\end{aligned}$$

$$\begin{aligned}
C(i,j+1) &= \frac{\Delta t}{Sc(\Delta x)^2}\left[C(i-1,j)-2C(i,j)+C(i+1,j)\right] \\
&- \frac{S\Delta t}{\Delta x}C(i+1,j)+\left\{\frac{S\Delta t}{\Delta x}+1\right\}C(i,j) \qquad (3.85)
\end{aligned}$$

3.2.4 Observations and discussions

Computations for the velocities u and w, temperature θ and concentration C were made for Pr = 0.71, Gr = -0.5 (corresponding to heating), Gr =0.5 (corresponding to cooling), Gc =1.5 and $M^2 = 6.0$. The parameters that were varied included the time t, Hall parameter m, suction parameter S, Schmidt number Sc and the Ekcert number Ec. The reference values represented in figures 3.5 on page 62 to figure 3.12 on page 68 by the curve labeled “test ” are t = 3, m = 1.0, S =

-0.25, Sc = 0.4 and Ec = 0.1. These values of the parameters were varied one at a time and input into a C++ computer program. Computations were done using the simultaneous equations (3.82) to (3.85), the initial conditions (3.80) and the boundary conditions (3.81) and curves plotted for each case. These results for the case of heating at the plate (Gr = -0.5) were then represented in the figures labelled figure 3.5 to figure 3.8 and the results for the case of cooling at the plate (Gr = 0.5) are represented in the figures labelled figure 3.9 on page 66 to figure 3.12 on page 68. The horizontal axis for the figures 3.5 to figure 3.12 represents the distance from the left plate.

(a) Primary velocity profiles with heating

The effects of various parameters on the primary velocity of the fluid flow were considered as discussed below with reference to figure 3.5 on page 62.

Suction

When there is no suction (S = 0), there is a decrease in the fluid velocity profiles.

Schmidt number

An increase in the value of the Schmidt number (Sc = 0.7), leads to a slight decrease in the fluid velocity profiles. This is because the mass diffusion parameter is directly proportional to the shear stresses and therefore its increase would result in retarding fluid motion.

Eckert number

As the value of the Eckert number Ec increases (Ec = 0.2), the primary velocity decreases slightly.

Hall parameter

As the value of the Hall parameter m increases (m = 2.0), the primary velocity increases slightly. This can be attributed to the effect of an increase in the cyclotron frequency and thus the rotation of the electrons and/or an increase in the collision times of the electrons with an increase in the value of the Hall parameter. An increase in the Hall parameter leads to a decrease in the effective conductivity $\frac{\sigma}{1+m^2}$ which reduces the magnetic dumping force on the velocity and consequently the velocity increases.

Time

A decrease in the time t leads to a decrease in the primary velocity profiles. This is because the time is directly proportional to the primary velocity.

(b) Secondary velocity profiles with heating

Generally, the secondary velocity increases as the moving plate is approached and then soon after reduces to zero at the moving plate.

Suction

Removal of suction leads to a decrease in the secondary velocity profiles.

Eckert number

An increase in the Eckert number leads to a decrease in the secondary velocity profiles.

Schmidt number

An increase in the Schmidt number leads to a decrease in the secondary velocity profiles. This is because the mass diffusion parameter is directly proportional to the shear stresses and therefore its increase would result in retarding fluid velocity.

Hall parameter

An increase in the Hall parameter leads to a decrease in the secondary velocity profiles.

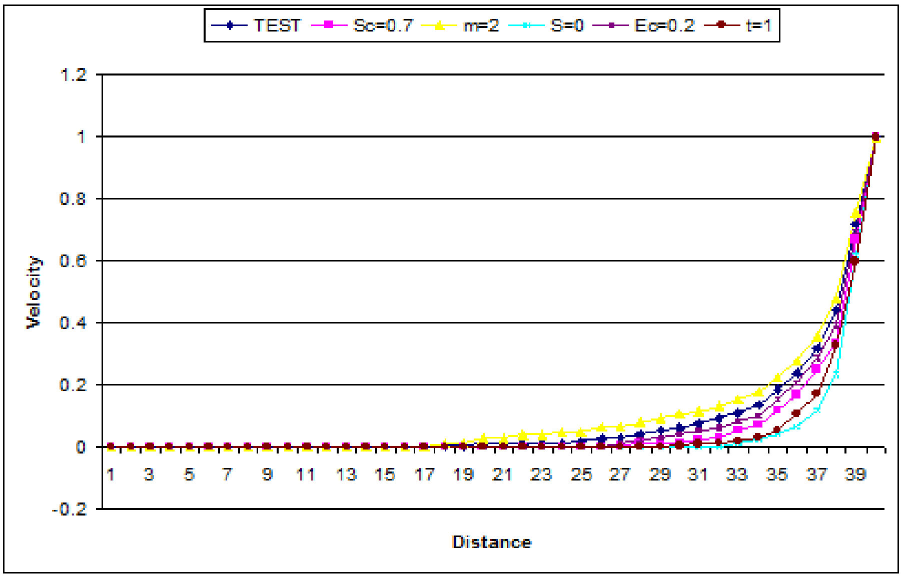

FIGURE 3.5. Primary velocity profiles for vertical plates with one plate moving. Gr =-0.5

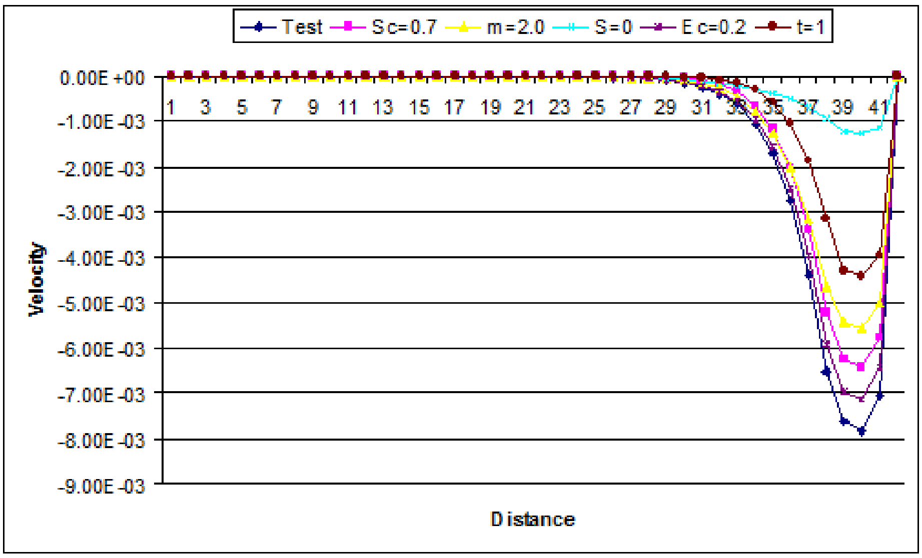

FIGURE 3.6. Secondary velocity profiles for vertical plates with one plate moving. Gr = -0.5

Time

A decrease in the time leads to a decrease in the secondary velocity profiles. This is because the time is directly proportional to the secondary velocity.

(c) Temperature profiles with heating

The various parameters affected the temperature profiles of the fluid flow as discussed below.

Eckert number An increase in the Eckert number leads to an increase in the temperature profiles. The rate of increase is however slower as the moving plate is approached. This is due to the fact that an increase in the Eckert number means a decrease in the thermal energy and consequently a decrease in the temperature of the fluid which leads to a decrease in the temperature profiles.

Suction parameter Removal of suction leads to a decrease in the temperature profiles since this removes the influence of convection.

Hall parameter An increase in the Hall parameter leads to a very slight decrease in the temperature profiles. This is due to a decrease in the Joule dissipation.

Time A decrease in the time leads to a decrease in the temperature profiles. This is because the time is directly proportional to the temperature.

(d) Concentration profiles with heating

Schmidt number

An increase in the Schmidt number (Sc = 0.7), lead to a decrease in the concentration profiles. The mass diffusion parameter Sc is inversely proportional to the concentration and therefore its increase results in a decrease in the concentration profiles.

Suction

Removal of suction (S = 0) leads to a decrease in the concentration profiles.

Time A decrease in the time leads to a decrease in the concentration profiles. This is because the time is directly proportional to the concentration.

(e) Primary velocity profiles with cooling

The effects of various parameters on the primary velocity of the fluid flow were considered as discussed below with reference to figure 3.9 on page 66.

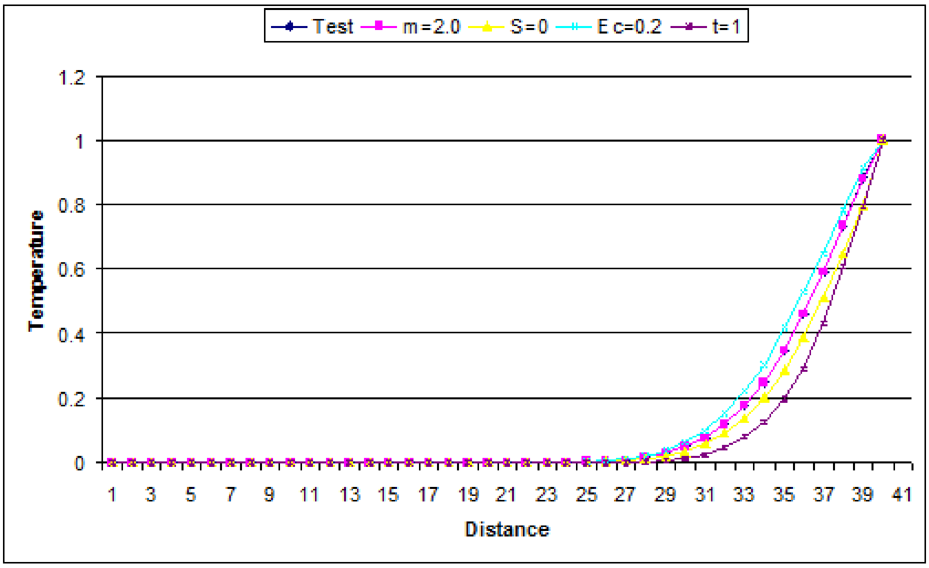

FIGURE 3.7. Temperature profiles for vertical plates with one plate moving. Gr = -0.5

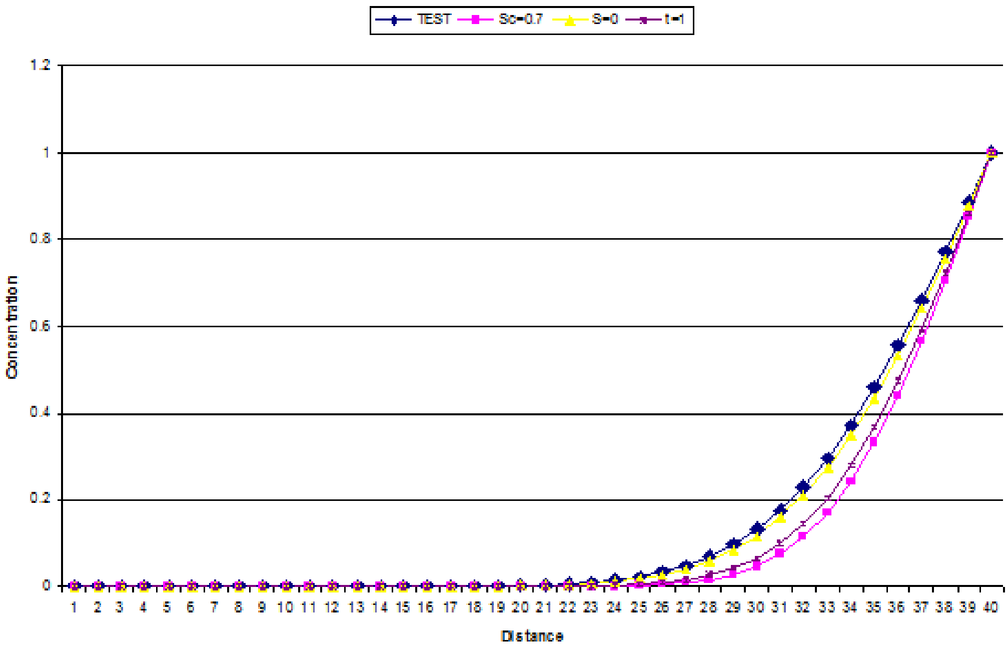

FIGURE 3.8. Concentration profiles for vertical plates with one plate moving. Gr= -0.5

Eckert number An increase in the Eckert number leads to a slight decrease in the primary velocity profiles.

Suction parameter Removal of suction leads to a decrease in the primary velocity profiles.

Schmidt number An increase in the Schmidt number leads to a decrease in the primary velocity profiles. This is because the mass diffusion parameter is directly proportional to the shear stresses and therefore its increase would result in retarding fluid motion.

Hall parameter An increase in the Hall parameter leads to an increase in the primary velocity profiles. This is due to a decrease in the effective conductivity $\frac{\sigma}{1+m^2}$ which reduces the magnetic dumping force on the velocity and consequently the velocity increases.

Time A decrease in the time leads to a decrease in the primary velocity profiles. This is because the time is directly proportional to the primary velocity.

(f) Secondary velocity profiles with cooling

The secondary velocity increases as the moving plate is approached and then soon after reduces to zero at the moving plate. Considering figure 3.10 on the next page we make the following observations.

Eckert number An increase in the Eckert number leads to an increase in the secondary velocity profiles. This is due to the fact that an increase in the Eckert number means an increase in the kinetic energy and consequently an increase in the fluid velocities which leads to an increase in the velocity profiles.

Suction parameter Removal of suction leads to a decrease in the secondary velocity profiles.

Schmidt number An increase in the Schmidt number leads to a decrease in the secondary velocity profiles. This is because the mass diffusion parameter is directly proportional to the shear stresses and therefore its increase would result in retarding fluid velocity.

Hall parameter An increase in the Hall parameter leads to a decrease in the secondary velocity profiles.

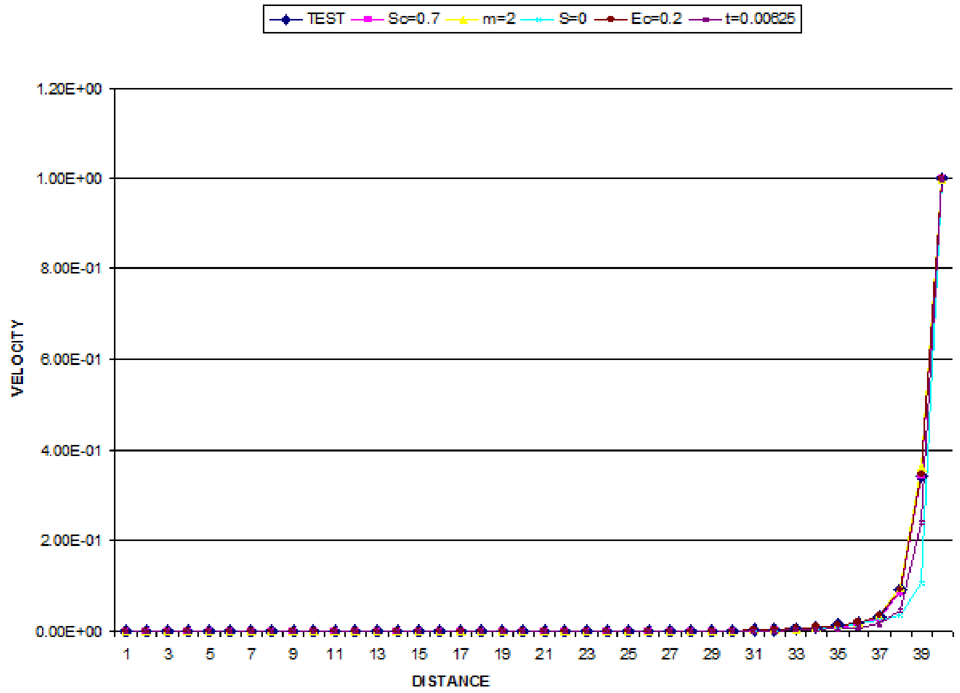

FIGURE 3.9. Primary velocity profiles for vertical plates with one plate moving. Gr =0.5

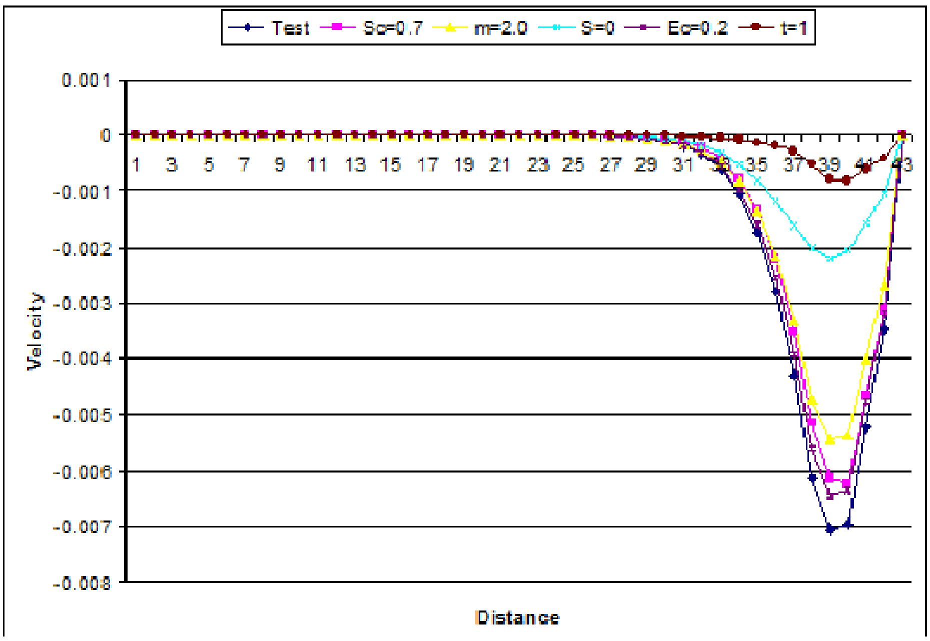

FIGURE 3.10. Secondary velocity profiles for vertical plates with one plate moving. Gr =0.5

Time A decrease in the time leads to a decrease in the secondary velocity profiles. This is because the time is directly proportional to the secondary velocity.

(g) Temperature profiles with cooling

The various parameters affected the temperature profiles of the fluid flow as discussed below with reference to figure 3.11 on the following page.

Eckert number An increase in the Eckert number leads to an increase in the temperature profiles. The rate of increase is however slower as the moving plate is approached.

Suction parameter Removal of suction leads to a decrease in the temperature profiles.

Schmidt number An increase in the Schmidt number leads to an increase in the temperature profiles.

Hall parameter An increase in the Hall parameter leads to a slight decrease in the temperature profiles. This is because an increase in the Hall parameter causes a decrease in the Joule dissipation which in turn leads to a decrease in the temperature profiles.

Time A decrease in the time leads to a decrease in the temperature profiles. This is because the time is directly proportional to the temperature.

(h) Concentration profiles with cooling

The effect of changing the various parameters on the concentration profiles are discussed below with reference to figure 3.12 on the next page.

Suction parameter Removal of suction leads to a decrease in the concentration profiles.

Schmidt number An increase in the Schmidt number leads to a decrease in the concentration profiles. The mass diffusion parameter Sc is inversely proportional to the concentration and therefore its increase results in a decrease in the concentration profiles.

Time A decrease in the time leads to a decrease in the concentration profiles. This is because the time is directly proportional to the concentration.

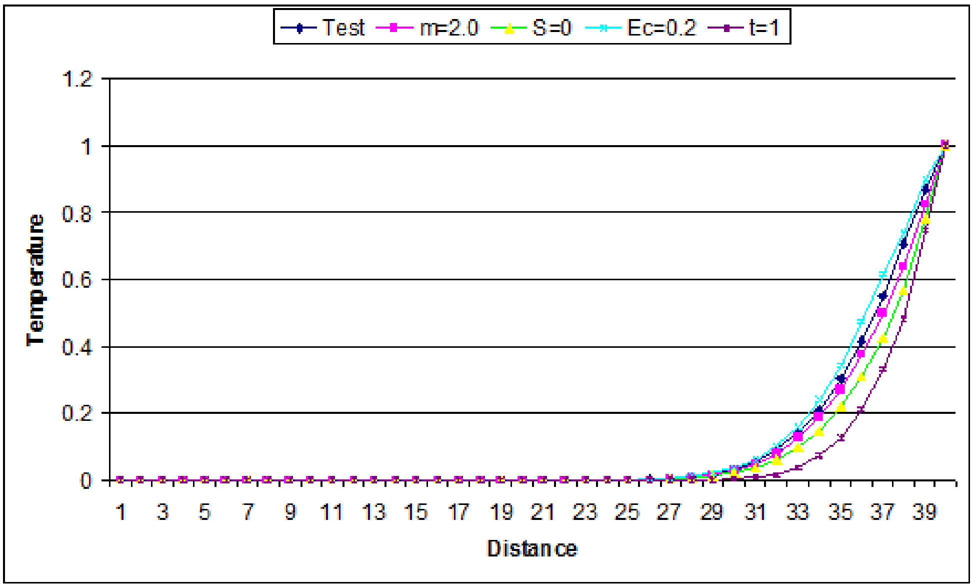

FIGURE 3.11. Temperature profiles for vertical plates with one plate moving. Gr = 0.5

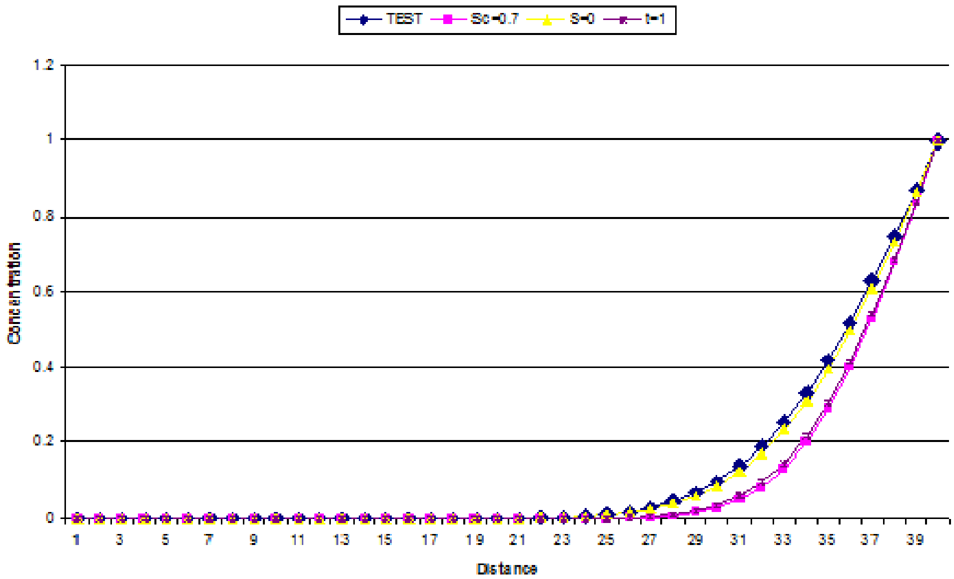

FIGURE 3.12. Concentration profiles for vertical plates with one plate moving. Gr = 0.5

Chapter 4

MHD FLUID FLOW BETWEEN TWO MOVING PARALLEL POROUS PLATES WITH EFFECT OF HALL CURRENT

In this chapter, we consider the unsteady MHD fluid flow between two parallel plates with heat transfer while putting into consideration the effect of the Hall current. We further consider the two plates to be moving unlike the case of chapter three where only one plate was moving. There are three cases that are considered. One is the case where the fluid flows between horizontal parallel plates, then a case is considered when the system is rotating and the third case has the fluid flowing between two vertical parallel plates.

4.1 Fluid flow between two parallel horizontal plates with both plates moving

Consider an incompressible, viscous, heat conducting and electrically conducting fluid flowing between two infinite non conducting porous horizontal plates. The fluid flow is unsteady and a magnetic field is applied perpendicular to the plates. The upper plate is moving with a constant velocity U_0 while the lower plate is moving with a velocity $-U_0$. The induced magnetic field is neglected by assuming a very small magnetic Reynold's number. For this reason, the uniform magnetic field B_0 is considered as the total magnetic field acting on the fluid. The effect of the Hall current and the effects of suction and injection are put into consideration while studying the fluid flow in this current chapter.

4.1.1 Mathematical formulation

A description of the fluid and the fluid flow being studied is given and then a Mathematical formulation of the fluid flow problem given. Let the two horizontal plates that are electrically non conducting be located on the planes $y = \pm h$. The plates are of infinite length in the x and z directions i.e. $-\infty < x < \infty$ and $-\infty < z < \infty$. We consider the fluid to be flowing between the two plates under the influence of a constant pressure gradient $\frac{dP}{dx}$ in the x direction, and a uniform injection from below and suction from above with a constant velocity v_0. The pressure gradient is applied at time t = 0. The upper plate is initially at rest but is set into motion at time t = 0. A uniform magnetic field B_0 acts on the whole system in the positive y direction as shown in figure 4.1 below.

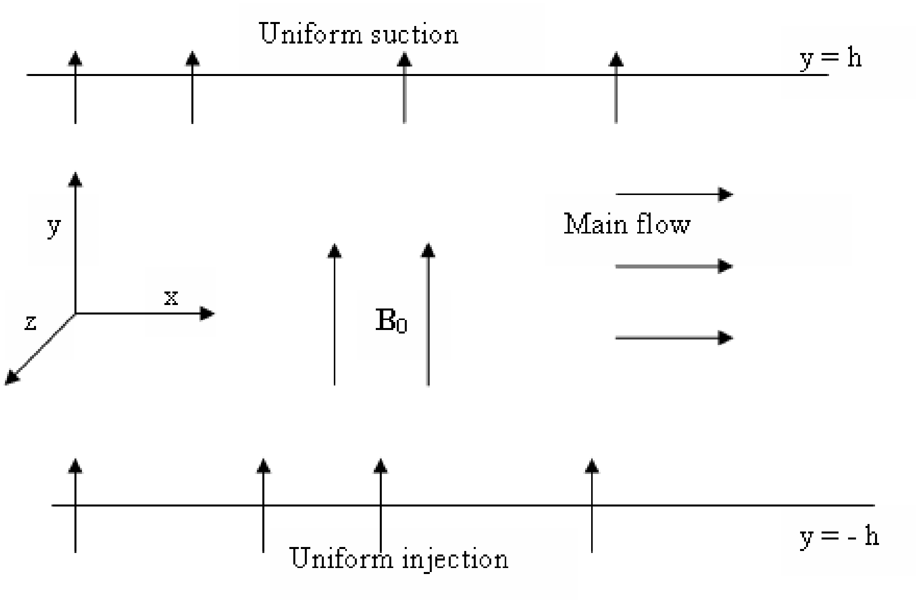

FIGURE 4.1. Fluid flow between two moving horizontal plates with Hall effect

For the fluid flow being considered, all quantities depend on the space coordinate y and time t except the pressure gradient $\frac{dP}{dx}$ which is assumed constant. The velocity of the fluid is given as $\mathbf{q}(y,t) = u(y,t)\mathbf{i} + v_0\mathbf{j} + w(y,t)\mathbf{k}$ and the applied magnetic field acts along the y axis and is given by $\boldsymbol{B}_0 = \langle 0, B_0, 0\rangle$. When the strength of the magnetic field is large, Ohm's law is modified to include the Hall currents. The horizontal plates are considered to be moving in opposite directions relative to each other. In general the fluid flow configuration in this chapter has much similarity with that of chapter three save for the fact that in this chapter, both plates are considered to be moving.

4.1.2 Governing equations in dimensional and non dimensional form

The conservation of momentum equations are written in component form as the equations (3.11) and (3.12) of chapter three. This is because the geometry of both systems of chapter three and this current chapter is the same. The energy conservation equation with consideration of the viscous and the Joule dissipation can be expressed as equation (3.17) of chapter three and the concentration conservation equation is given as equation (3.18) of chapter three. The dimensional form of these equations is therefore given below by equations (4.1) to equation (4.4).

$$\frac{\partial u^*}{\partial t^*} + v_0^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{dP}{dx} + \upsilon\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(u^* + mw^*) \tag{4.1}$$

$$\frac{\partial w^*}{\partial t^*} + v_0^*\frac{\partial w^*}{\partial y^*} = \upsilon\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mu^* - w^*) \tag{4.2}$$

$$\begin{aligned}\rho C_p\left(\frac{\partial T^*}{\partial t^*} + v_0^*\frac{\partial T^*}{\partial y^*}\right) &= K\frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left(\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right)\\ &+ \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u^{*2} + w^{*2})\end{aligned} \tag{4.3}$$

$$\frac{\partial C^*}{\partial t^*} + v_0^*\frac{\partial C^*}{\partial y^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{4.4}$$

The first two equations give the momentum conservation equations along the x and z axes respectively. Equation (4.3) is the energy conservation equation while equation (4.4) is the concentration equation. For the fluid flow in consideration, let the two plates be initially isothermal at temperature T_1^*. The temperature of the upper plate is then raised to T_2^* and thereafter the lower and upper plates are maintained at two different but constant temperatures T_1^* and T_2^* respectively where $T_1^*<T_2^*$. The plates are initially at rest but at time $t = 0$ the upper plate is set into motion with a velocity U_0 and the lower plate is also set into motion with the same velocity but in the opposite direction. Let the concentration at both plates be C_1^* initially. The concentration at the upper plate is then increased to C_2^* at time $t = 0$ and the concentrations at each of the boundaries are kept constant thereafter. From the definition of the fluid flow problem under consideration, the

initial and boundary conditions are respectively given below as

$$\left.\begin{array}{l} u^*(-h^*,0)=w^*(-h,0)=0 \\ u^*(h^*,0)=w^*(h,0)=0 \\ T^*(-h,0)=T^*(h,0)=T_1^* \\ C^*(-h,0)=C^*(h,0)=C_1^* \end{array}\right\} \quad t^*=0 \tag{4.5}$$

$$\left.\begin{array}{l} u^*(-h^*,t^*)=-U_0 \\ w^*(-h^*,t^*)=0 \\ u^*(h^*,t^*)=U_0 \\ w^*(h^*,t^*)=0 \\ T^*(-h^*,t^*)=T_1^* \\ T^*(h^*,t^*)=T_2^* \\ C^*(-h^*,t^*)=C_1^* \\ C^*(h^*,t^*)=C_2^* \end{array}\right\} \quad t^*>0 \tag{4.6}$$

The non dimensional form of the momentum conservation equations, the energy equation and the mass conservation equation is given from equations (3.26), (3.27), (3.33) and (3.34) of chapter three as equations (4.7) to equation (4.10) below.

$$\frac{\partial u}{\partial t}+S\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{\partial^2 u}{\partial y^2}-\frac{M^2}{1+m^2}(u+mw) \tag{4.7}$$

$$\frac{\partial w}{\partial t}+S\frac{\partial w}{\partial y}=\frac{\partial^2 w}{\partial y^2}+\frac{M^2}{1+m^2}(mu-w) \tag{4.8}$$

$$\frac{\partial \theta}{\partial t}+S\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2}+Ec\frac{\partial \boldsymbol{q}}{\partial y}\cdot\frac{\partial \bar{\boldsymbol{q}}}{\partial y}+\frac{EcM^2}{1+m^2}\mathbf{q}\bar{\mathbf{q}} \tag{4.9}$$

$$\frac{\partial C}{\partial t}+S\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{4.10}$$

The non dimensional forms of the initial conditions (4.5) are given as (4.11)and the boundary conditions in the equation (4.6) are given in non dimensional form as (4.12)

$$\left.\begin{array}{l} q(-L,0)=q(L,0)=0 \\ \theta(-L,0)=\theta(L,0)=0 \\ C(-L,0)=C(L,0)=0 \end{array}\right\} \quad t=0 \tag{4.11}$$

$$\left.\begin{array}{l} q(-L,t)=-1 \\ q(L,t)=1 \\ \theta(-L,t)=0 \\ \theta(L,t)=1 \\ C(-L,t)=0 \\ C(L,t)=1 \end{array}\right\} \quad t>0 \tag{4.12}$$

4.1.3 Solution

We seek the solution of the system of equations (4.7),(4.8),(4.9) and (4.10) together with the initial conditions (4.11) and the boundary conditions (4.12). The system of equations is non linear and we apply the numerical approximation method of finite differences in the solution. Using forward differences, the finite difference form of equations governing the fluid flow under consideration i.e. equations (4.7),(4.8),(4.9) and (4.10) have the form

$$\begin{aligned}\frac{u(i,j+1)-u(i,j)}{\Delta t} &= -\frac{dP}{dx}+\frac{u(i-1,j)-2u(i,j)+u(i+1,j)}{(\Delta y)^2}\\ &- S\left\{\frac{u(i+1,j)-u(i,j)}{\Delta y}\right\}\\ &- \frac{M^2}{1+m^2}\left[u(i,j)+mw(i,j)\right]\end{aligned} \tag{4.13}$$

$$\begin{aligned}\frac{w(i,j+1)-w(i,j)}{\Delta t} &= \frac{w(i-1,j)-2w(i,j)+w(i+1,j)}{(\Delta y)^2}\\ &- S\left\{\frac{w(i+1,j)-w(i,j)}{\Delta y}\right\}\\ &+ \frac{M^2}{1+m^2}\left[mu(i,j)-w(i,j)\right]\end{aligned} \tag{4.14}$$

$$\begin{aligned}\frac{\theta(i,j+1)-\theta(i,j)}{\Delta t} &= \frac{1}{Pr}\left[\frac{\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)}{(\Delta y)^2}\right]\\ &- S\left\{\frac{\theta(i+1,j)-\theta(i,j)}{\Delta y}\right\}\\ &+ Ec\left[\left(\frac{u(i+1,j)-u(i,j)}{\Delta y}\right)^2+\left(\frac{w(i+1,j)-w(i,j)}{\Delta y}\right)^2\right]\\ &+ \frac{EcM^2}{1+m^2}\left[u(i,j)^2+w(i,j)^2\right]\end{aligned} \tag{4.15}$$

$$\begin{aligned}\frac{C(i,j+1)-C(i,j)}{\Delta t} &= \frac{1}{Sc}\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta y)^2}\right]\\ &- S\left[\frac{C(i+1,j)-C(i,j)}{\Delta y}\right]\end{aligned} \tag{4.16}$$

The finite difference form of the initial conditions (4.11) and the boundary conditions (4.12) is given below.

$$\left.\begin{array}{l} q(-40,0)=q(40,0)=0 \\ \theta(-40,0)=\theta(40,0)=0 \\ C(-40,0)=C(40,0)=0 \end{array}\right\} \quad j=0 \tag{4.17}$$

$$\left.\begin{array}{l} q(-40,j)=-1 \\ q(40,j)=1 \\ \theta(-40,j)=0 \\ \theta(40,j)=1 \\ C(-40,j)=0 \\ C(40,j)=1 \end{array}\right\} \quad j>0 \tag{4.18}$$

In this case i and j represent the space coordinate y and the time coordinate t respectively. Rearranging each of these equations enables us to compute consecutive terms of the primary velocity u, the secondary velocity w, the temperature θ and the concentration C using the initial values and boundary conditions given in equations (4.17) and (4.18). Rearrangement of the equations (4.13) to (4.16) gives equations (4.19) to (4.22) below. The solutions to these equations together with the initial conditions (4.11) and boundary conditions (4.12) are computed using an appropriate computer program. A C++ computer program is written and used for the solution. The method is stable, convergent and consistent.

$$\begin{aligned} u(i,j+1) &= -\Delta t\frac{dP}{dx}+\frac{\Delta t}{(\Delta y)^2}\left[u(i-1,j)-2u(i,j)+u(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta y}\left[u(i+1,j)-u(i,j)\right]-\left[\frac{\Delta t M^2}{1+m^2}-1\right]u(i,j) \\ &- \Delta t\left[\frac{mM^2}{1+m^2}\right]w(i,j) \end{aligned} \tag{4.19}$$

$$\begin{aligned} w(i,j+1) &= \frac{\Delta t}{(\Delta y)^2}\left[w(i-1,j)-2w(i,j)+w(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta y}\left[w(i+1,j)-w(i,j)\right] \\ &- \left[\frac{\Delta t M^2}{1+m^2}-1\right]w(i,j)+\Delta t\left[\frac{mM^2}{1+m^2}\right]u(i,j) \end{aligned} \tag{4.20}$$

$$
\begin{aligned}
\theta(i,j+1) &= \frac{\Delta t}{\Pr(\Delta y)^2}\left[\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)\right]-S\frac{\Delta t}{\Delta y}\theta(i+1,j)\\
&+ Ec\frac{\Delta t}{(\Delta y)^2}\left[\{u(i+1,j)-u(i,j\}^2+\{w(i+1,j)-w(i,j\}^2\right]\\
&+ \left[S\frac{\Delta t}{\Delta y}+1\right]\theta(i,j)+Ec\frac{M^2\Delta t}{1+m^2}\left[u(i,j)^2+w(i,j)^2\right] \qquad (4.21)
\end{aligned}
$$

$$
\begin{aligned}
C(i,j+1) &= \frac{\Delta t}{Sc(\Delta y)^2}\left[C(i-1,j)-2C(i,j)+C(i+1,j)\right]-\frac{S\Delta t}{\Delta y}C(i+1,j)\\
&+ \left\{\frac{S\Delta t}{\Delta y}+1\right\}C(i,j) \qquad (4.22)
\end{aligned}
$$

4.1.4 Calculation of the rate of mass transfer, skin friction and the rate of heat transfer

To calculate the skin friction, the rate of heat transfer and the rate of mass transfer we consider a quadratic bivariate polynomial which is a function of distance (y) and time (t). The second degree bivariate polynomials approximating the primary velocity u, secondary velocity w, temperature θ , and concentration C in the variables y and t are given below

$u'(y,t) = a_1 + b_1 y + c_1 t + d_1 y^2 + e_1 t^2 + f_1 yt$

$v'(y,t) = a_2 + b_2 y + c_2 t + d_2 y^2 + e_2 t^2 + f_2 yt$

$\theta'(y,t) = a_3 + b_3 y + c_3 t + d_3 y^2 + e_3 t^2 + f_3 yt$

$C'(y,t) = a_4 + b_4 y + c_4 t + d_4 y^2 + e_4 t^2 + f_4 yt$

We are going to use the least squares approximation method discussed in section 2.3.3 on page 33. The skin friction on the lower plate in the direction of the x axis is obtained as

$$
\begin{aligned}
\tau_x &= -\left.\frac{\partial u}{\partial y}\right|_{y=-l} = -(b_1+2d_1 y+f_1 t)|_{y=-l}\\
&= -(b_1-2d_1 l+f_1 t) \qquad (4.23)
\end{aligned}
$$

while the skin friction on the upper boundary in the same direction is given by

$$
\begin{aligned}
\tau_x &= -\left.\frac{\partial u}{\partial y}\right|_{y=l} = -(b_1+2d_1 y+f_1 t)|_{y=l}\\
&= -(b_1+2d_1 l+f_1 t) \qquad (4.24)
\end{aligned}
$$

Similarly, the skin friction on the lower plate in the direction of the z axis is obtained as

$$\begin{aligned} \tau_z &= -\left.\frac{\partial w}{\partial y}\right|_{y=-l} = -(\mathrm{b}_2 + 2\mathrm{d}_2\mathrm{y} + \mathrm{f}_2\mathrm{t})|_{y=-l} \\ &= -(\mathrm{b}_2 - 2\mathrm{d}_2\mathrm{l} + \mathrm{f}_2\mathrm{t}) \end{aligned} \tag{4.25}$$

while the skin friction on the upper boundary in the same direction is given by

$$\begin{aligned} \tau_z &= -\left.\frac{\partial w}{\partial y}\right|_{y=l} = -(\mathrm{b}_2 + 2\mathrm{d}_2\mathrm{y} + \mathrm{f}_2\mathrm{t})|_{y=l} \\ &= -(\mathrm{b}_2 + 2\mathrm{d}_2\mathrm{l} + \mathrm{f}_2\mathrm{t}) \end{aligned} \tag{4.26}$$

The rate of heat transfer on the lower plate is obtained as

$$\begin{aligned} Nu &= -\left.\frac{\partial \theta}{\partial y}\right|_{y=-l} = -(\mathrm{b}_3 + 2\mathrm{d}_3\mathrm{y} + \mathrm{f}_3\mathrm{t})|_{y=-l} \\ &= -(\mathrm{b}_3 - 2\mathrm{d}_3\mathrm{l} + \mathrm{f}_3\mathrm{t}) \end{aligned} \tag{4.27}$$

while the rate of heat transfer on the upper boundary in the same direction is given by

$$\begin{aligned} Nu &= -\left.\frac{\partial \theta}{\partial y}\right|_{y=l} = -(\mathrm{b}_3 + 2\mathrm{d}_3\mathrm{y} + \mathrm{f}_3\mathrm{t})|_{y=l} \\ &= -(\mathrm{b}_3 + 2\mathrm{d}_3\mathrm{l} + \mathrm{f}_3\mathrm{t}) \end{aligned} \tag{4.28}$$

The rate of mass transfer is given by

$$\begin{aligned} Sh &= -\left.\frac{\partial C}{\partial y}\right|_{y=-l} = -(\mathrm{b}_4 + 2\mathrm{d}_4\mathrm{y} + \mathrm{f}_4\mathrm{t})|_{y=-l} \\ &= -(\mathrm{b}_4 - 2\mathrm{d}_4\mathrm{l} + \mathrm{f}_4\mathrm{t}) \end{aligned} \tag{4.29}$$

while the rate of mass transfer on the upper boundary in the same direction is given by

$$\begin{aligned} Sh &= -\left.\frac{\partial C}{\partial y}\right|_{y=l} = -(\mathrm{b}_4 + 2\mathrm{d}_4\mathrm{y} + \mathrm{f}_4\mathrm{t})|_{y=l} \\ &= -(\mathrm{b}_4 + 2\mathrm{d}_4\mathrm{l} + \mathrm{f}_4\mathrm{t}) \end{aligned} \tag{4.30}$$

The equations (4.23) to (4.30) are used to obtain values of the skin friction, the rate of heat transfer as well as the rate of mass transfer on the flow field for various values of the rotation parameter Er , Schmidt number Sc, Hall parameter m, Suction parameter S, Eckert number Ec, Pressure gradient $\frac{dP}{dx}$ and time t.

4.1.5 Observations and discussions

Computations for the primary velocity u, secondary velocity w, temperature θ and concentration C were made for various parameters with Pr = 0.71 and $M^2 =$ 6.0. The parameters that were varied included the pressure gradient $\frac{dP}{dx}$, the Hall parameter m, the suction parameter S, the Schmidt number Sc and the Ekcert number Ec. The reference values represented in figures 4.2 on page 79 to 4.9 on page 85 by the curve labeled "test" are $\frac{dP}{dx}$ = 5.0 , m = 1.0, S = - 0.5, Sc = 0.4 and Ec = 0.2. These values of the parameters were varied one at a time and input into a computer program. Computations were done using equations (4.19) to (4.22), the initial conditions (4.11) and the boundary conditions (4.12) and curves were plotted for each case. These results were then represented in the figures labelled figure 4.2 on page 79 to figure 4.9 on page 85.

The values of velocities, temperature, and concentration obtained in section 4.1.3 are used for different values of the rotation parameter Er, Schmidt number Sc, Hall parameter m, Suction parameter S, Eckert number Ec, Pressure gradient $\frac{dP}{dx}$ and time t are used to obtain the values of the skin friction and the rates of heat and mass transfer. The Tables represent the variation in the rates of heat and mass transfer as well as the skin friction on the laminar thermal, concentration and velocity boundary layers respectively. The reference values used for the parameters are Er = 0.05 for the rotation parameter, Sc = 0.4 for the Schmidt number, m = 1.0 for the Hall parameter, S = -0.5 for the Suction parameter, Ec = 0.2 for the Eckert number, $\frac{dP}{dx}$ = 5.0, for the pressure gradient and t = 0.25 for the time. Each of the parameters is varied and the results are presented in tables 4.1 on page 86 to table 4.4 on page 86. The observations made from these tables are then discussed after discussing the concentration profiles.

a) Primary velocity profiles

We discuss how each of the parameters affects the primary velocity profiles of the fluid flow as represented by the graphs in figure 4.2 on page 79. For clarity, the profiles near the lower plate are shown in figure 4.3 on page 79 and the profiles near the upper plate are shown in figure 4.4 on page 81.

Pressure gradient

From figures 4.2 on the following page, figure 4.3 on the next page and figure 4.4 on page 81 it is observed that very close to the lower boundary, both a positive and negative pressure gradient lead to a reduced fluid flow velocity until a free stream velocity is reached. However, a negative pressure gradient aids the fluid flow for a small region before the free stream velocity is achieved. It is observed that close to the lower plate, the primary velocity decreases but then soon after the velocity is maintained at a constant level except for the negative pressure gradient as discussed above. As the upper boundary is approached, the velocity in the case of positive pressure gradient reduces up to a point when u = 0 (stationary point) and then it increases suddenly until it reaches the velocity of the upper plate. For a negative pressure gradient, the velocity increases further from the free stream velocity to that of the upper plate as this plate is approached. The pressure gradient force term is proportional to the acceleration but act in opposite directions and therefore a positive pressure gradient retards the acceleration and consequently the velocity while a negative pressure gradient aids the flow.

Suction

When there is no suction ($S = 0$), the fluid velocity is observed to reach a higher velocity in the negative direction. The velocity starts reducing from the free stream velocity up to stagnation point and then increases up to the velocity of the upper plate at a faster rate as observed in figure 4.4 on page 81. The velocity profiles are thus reduced near the upper plate.

Hall parameter

As the value of the Hall parameter m increases, figure 4.2 on the following page shows that the reduction in the velocity is less and the free stream velocity is higher in the negative direction. From figure 4.4 on page 81 it is observed that as the upper plate is approached after stagnation velocity is reached, the primary velocity is more for increased value of the Hall parameter. An increase in the Hall parameter leads to a decrease in the effective conductivity $\frac{\sigma}{1+m^2}$ which reduces the

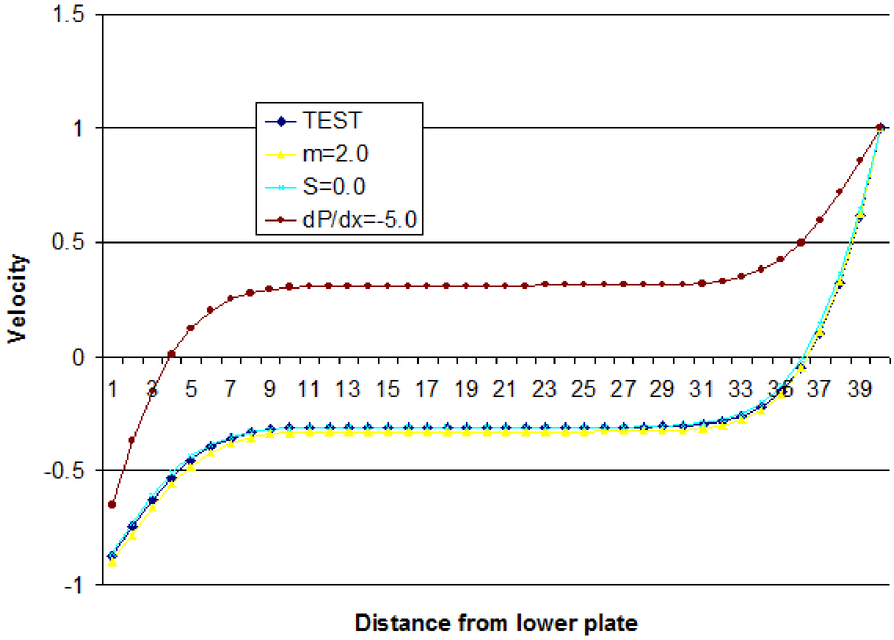

FIGURE 4.2. Primary velocity profiles for horizontal plates with both plates moving

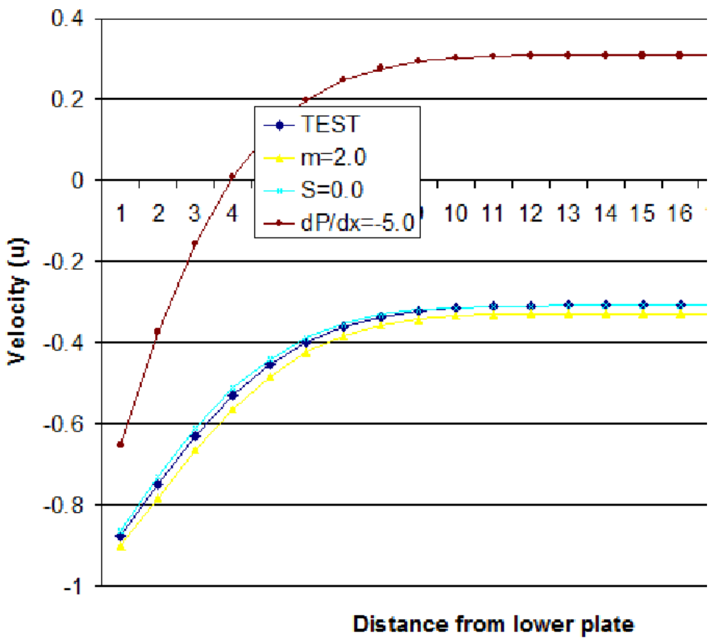

FIGURE 4.3. Primary velocity profiles near the lower plate for horizontal plates with both plates moving

magnetic dumping force on the velocity and consequently the velocity increases.

b) Secondary velocity profiles

We discuss how each of the parameters affects the secondary velocity profiles of the fluid flow. The reference graphs are found in figure 4.5 on the following page . It is observed that there is a flow in the negative direction near the lower plate for all cases. With the exception of the case of negative pressure gradient, the velocity is sustained in the reversed direction for the larger part of the flow region. The velocity then reduces, reaches a stagnation point and then increases in the positive direction up to a maximum and then reduces to zero at the upper plate.

Pressure gradient

Close to the lower plate, both a negative and positive pressure gradient lead to the fluid flowing in the negative direction. For a positive pressure gradient, a maximum is attained and then the velocity reduces to a constant velocity in the negative direction. The velocity then reduces further as the upper plate is approached, reaches a stagnation point and then increases to a maximum in the positive direction and then reduces to zero at the upper plate. For a negative pressure gradient, the velocity increases in the negative direction near the lower plate but soon after reduces to zero then increases to a lesser maximum in the positive direction. This is maintained for some time but as the upper plate is approached the velocity increases further to a higher maximum and then reduces to zero at the plate. The pressure gradient force term is proportional to the acceleration but act in opposite directions and therefore a positive pressure gradient retards the acceleration and consequently the velocity while a negative pressure gradient aids the flow.

Suction

Absence of suction leads to a similar behavior as in the case for a positive pressure gradient. However the maximum attained in the absence of suction is lower than the case when suction is considered to exist. This can be attributed to the effect of convection of the fluid in the presence of suction.

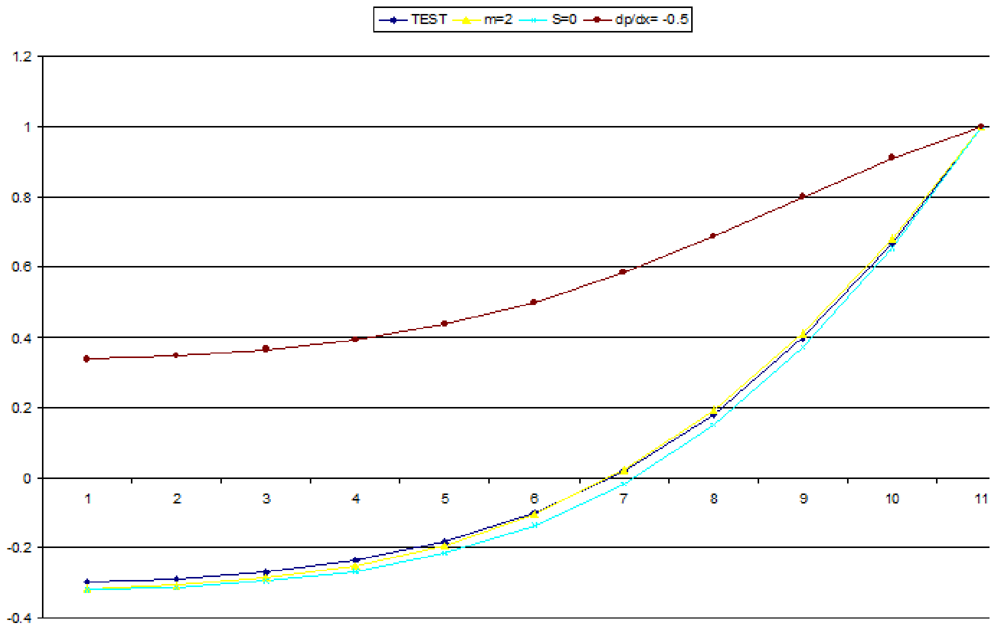

FIGURE 4.4. Primary velocity profiles near the upper plate for horizontal plates with both plates moving

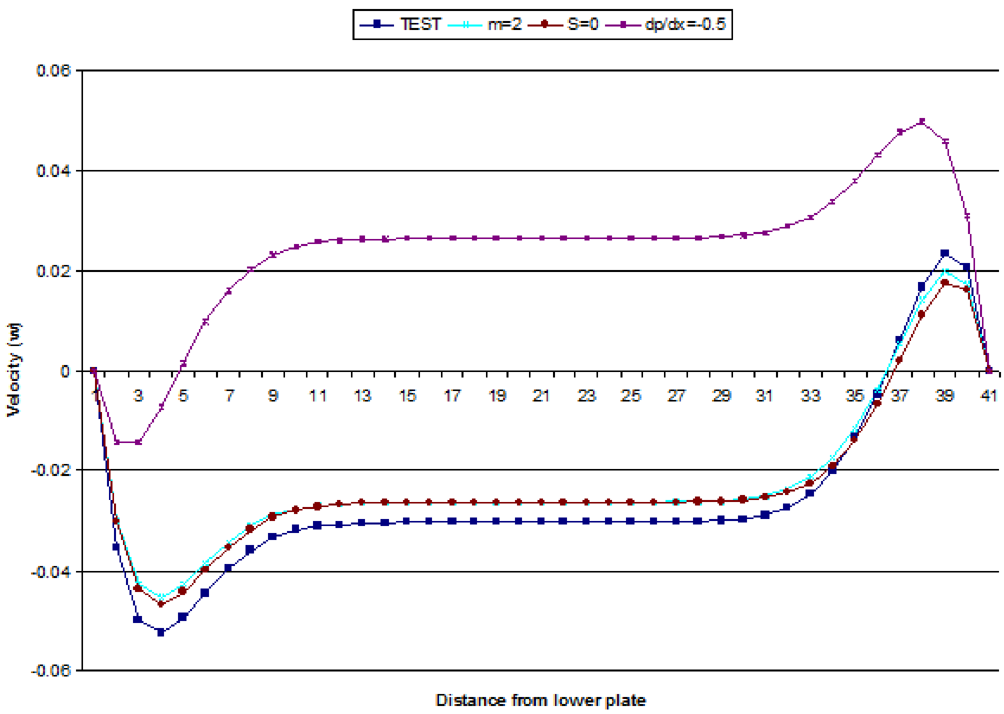

FIGURE 4.5. Secondary velocity profiles for horizontal plates with both plates moving

Hall parameter

An increase in the magnitude of the Hall parameter leads to a similar behavior as the case for a positive pressure gradient. However the maximum attained with an increase in the Hall parameter is lower than the case when a smaller value of the Hall parameter is considered.

c) Temperature profiles

A change in each of the parameters leads to a slight increase of the temperature profiles near the lower plate. This temperature then reduces to zero and then increases to the temperature of the upper plate as the upper plate is approached. Significant effects are observed as you get closer to the upper plate. Considering figure 4.6 on the next page, we discuss each of the parameters. To clearly see the profiles near the lower and upper plates consider respectively the figures 4.7 on the following page and figure 4.8 on page 85.

Pressure gradient

When the imposed pressure gradient is negative, the temperature increases slightly near the lower plate then reduces to zero as seen in figure 4.7 on the following page. The change is however not significant near the upper plate.

Hall parameter

An increase in the Hall parameter m leads to a lesser increase in the temperature profiles near the lower plate. There is a slight decrease in the temperature profiles near the upper plate. This is because an increase in the Hall parameter leads to a decrease in the Joule dissipation.

Suction

Absence of suction leads to the temperature increasing at a lesser extent near the lower plate. The rate of increase is more near the upper plate as observed from figure 4.8 on page 85. The temperature profiles are however reduced near the upper plate. This is due to the effect of convection.

Eckert number

A reduction in the value of the Eckert number Ec leads to a decrease in the temperature profiles near the plates. This is due to a decrease in the thermal

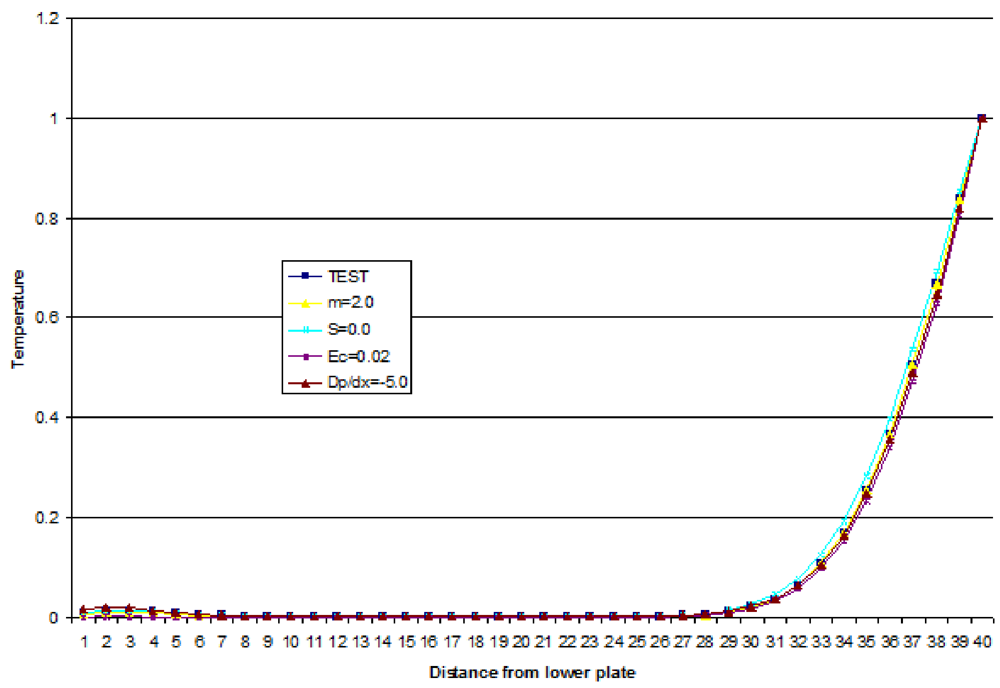

FIGURE 4.6. Temperature profiles for horizontal plates with both plates moving

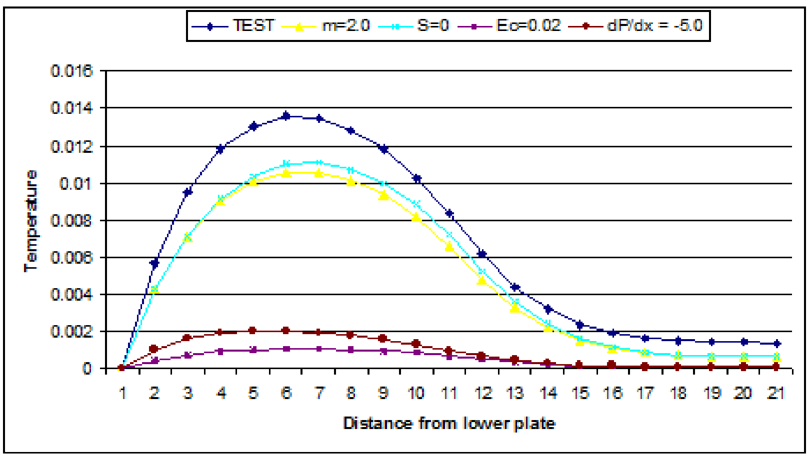

FIGURE 4.7. Temperature profiles near the lower plate for horizontal plates with both plates moving

energy and consequently a decrease in the temperature and hence the temperature profiles.

d) Concentration profiles

From figure 4.9 on the following page, it is observed that there is little effect produced by the change in the parameters near the lower plate but the effects are observed as you approach the upper plate.

Schmidt number

An increase in the value of the Schmidt number Sc causes the concentration to start increasing closer to the plate moving in the positive axis direction and at a faster rate. That is the concentration profiles reduce as the upper plate is approached. This is due to the fact that the mass diffusion parameter is inversely proportional to the the concentration and therefore its increase leads to a decrease in the concentration.

Suction

Removal of suction leads to concentration increase taking place closer to the plate moving in the positive axis direction and at a faster rate. The concentration profiles therefore reduce as the upper plate is approached.

Skin friction τ_x along the plates

The skin friction τ_x along the lower plate is positive while that along the upper plate is negative. We discuss the effect of changing each of the parameters on the skin friction τ_x with reference to the table 4.1 on page 86.

Hall parameter m An increase in the value of the Hall parameter $m, m = 2.0$ leads to a decrease in the magnitude of the skin friction τ_x along both the lower and upper plates.

Suction parameter S Removal of suction $S = 0$ leads to a decrease in the magnitude of the skin friction τ_x along both the lower and upper plates.

Pressure gradient A negative pressure gradient $\frac{dP}{dx}, \frac{dP}{dx} = -5.0$ leads to an increase in the magnitude of the skin friction τ_x along both the lower and upper plates.

Time t An increase in the time t, $t = 0.45$ leads to an increase in the magnitude

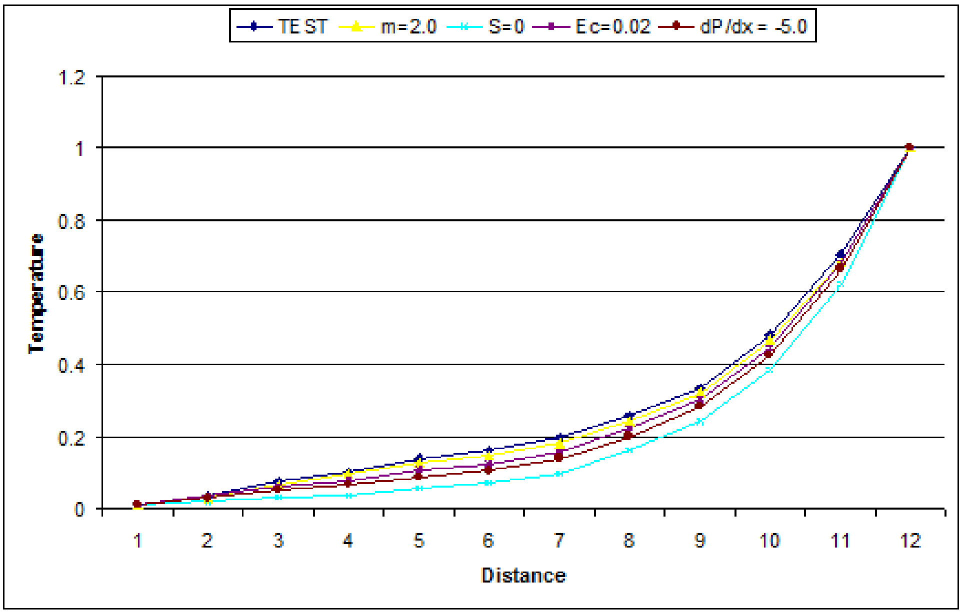

FIGURE 4.8. Temperature profiles near the upper plate for horizontal plates with both plates moving

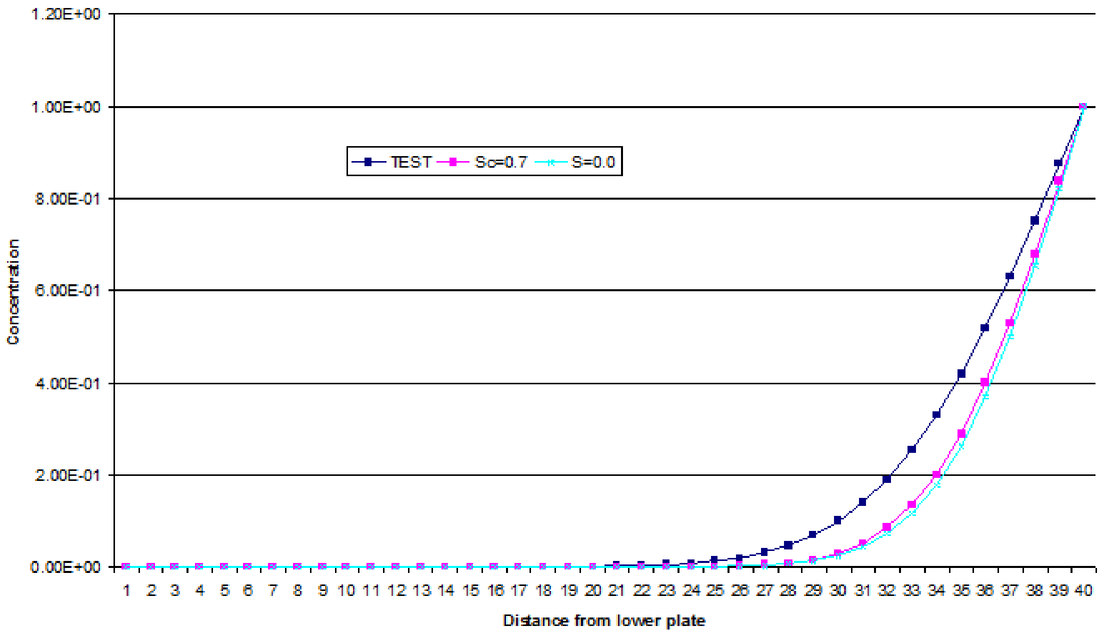

FIGURE 4.9. Concentration profiles for horizontal plates with both plates moving

						lower plate	upper plate
	m	S	Ec	$\frac{dP}{dx}$	t	τ_x	τ_x
Test	1	-0.5	0.2	5	0.25	261.2864556	-270.809491
2	2	-0.5	0.2	5	0.25	260.2735501	-269.7876455
3	2	0	0.2	5	0.25	258.5755013	-268.0806142
5	2	-0.5	0.02	-5	0.25	432.2208342	-443.8891401
6	2	-0.5	0.02	5	0.45	474.1248342	-483.6478696

TABLE 4.1. Skin friction along the plates for a fluid flow between two horizontal parallel plates

						lower plate	upper plate
	m	S	Ec	$\frac{dP}{dx}$	t	τ_y	τ_y
Test	1	-0.5	0.2	5	0.25	-3.362880657	0.5201359
2	2	-0.5	0.2	5	0.25	-3.637773285	0.798245045
3	2	0	0.2	5	0.25	-3.735169007	0.90310105
5	2	-0.5	0.02	-5	0.25	0.593629361	-3.562427826
6	1	-0.5	0.2	5	0.45	-4.916087279	2.073342522

TABLE 4.2. Skin friction perpendicular to two horizontal parallel plates

						lower plate	upper plate
	m	S	Ec	$\frac{dP}{dx}$	t	Nu	Nu
Test	1	-0.5	0.2	5	0.25	-5.642414138	2.806847198
2	2	-0.5	0.2	5	0.25	-5.368377224	2.52638825
3	2	0	0.2	5	0.25	-5.220497589	2.372087782
4	2	-0.5	0.02	5	0.25	-4.722799803	1.872184865
5	2	-0.5	0.02	-5	0.25	-4.879299423	2.032978756
6	1	-0.5	0.2	5	0.45	-9.022118672	6.186551732

TABLE 4.3. Rates of heat transfer for a fluid flow between two parallel horizontal plates

				lower plate	upper plate
	Sc	$\frac{dP}{dx}$	t	Sh	Sh
Test	0.4	5.0	0.25	-4.65178801	1.800214658
1	0.7	5.0	0.25	-4.651773002	1.800199624
6	0.4	5.0	0.45	-7.232589078	4.381015725

TABLE 4.4. Rates of mass transfer for a fluid flow between two parallel horizontal plates

of the skin friction τ_x along both the lower and upper plates.

Skin friction τ_z normal to the plates

The skin friction at the lower plate is negative while that at the upper plate is positive except for the negative pressure gradient. We discuss the effect of changing each of the parameters on the skin friction τ_z with reference to the table 4.2 on the preceding page.

Hall parameter m An increase in the value of the Hall parameter $m, m = 2.0$ leads to an increase in the magnitude of the skin friction τ_z at both the lower and upper plates.

Suction parameter S Removal of suction $S, S = 0$ leads to an increase in the magnitude of the skin friction τ_z at both the lower and upper plates.

Pressure gradient A negative pressure gradient leads to a reversal in the direction of the skin friction τ_z at both the lower and upper plates.

Time t An increase in the time t, $t = 0.45$ leads to an increase in the magnitude of the skin friction τ_z at both the lower and upper plates.

Rate of heat transfer Nu

The rate of heat transfer Nu is negative at the lower plate and positive at the upper plate. We discuss the effect of each of the parameters on the rate of heat transfer Nu with reference to the table 4.3 on the previous page.

Hall parameter m An increase in the value of the Hall parameter $m, m = 2.0$ leads to a decrease in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Suction parameter S Removal of suction $S, S = 0$ leads to a decrease in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Eckert number Ec A decrease in value of the Eckert number $Ec, Ec = 0.02$ leads to a decrease in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Pressure gradient A negative pressure gradient leads to an increase in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Time t An increase in the time t, $t = 0.45$ leads to an increase in the the magnitude of rate of heat transfer Nu at both the lower and upper plates.

Rate of mass transfer Sh

We use table 4.4 on page 86 to discuss the effect of changing each of the parameters on the rate of mass transfer.

Schmidt number Sc An increase in the Schmidt number Sc, $Sc = 0.7$ leads to a slight decrease in the magnitude of the rate of mass transfer Sh at both the lower and upper plates as observed from table 4.4 on page 86.

Time An increase in the time t, $t = 0.45$ leads to an increase in the rate of mass transfer Sh at both the lower and upper plates.

4.2 Fluid flow between two vertical parallel plates with both plates moving

The fluid flow that is considered in this section is unsteady, laminar and fully developed. The fluid is further taken to be incompressible, viscous, heat and electrically conducting and flows between two infinite non conducting porous vertical plates that are both moving in opposite directions relative to each other.

4.2.1 Mathematical formulation

Consider an unsteady, laminar, hydromagnetic fully developed fluid flowing between two parallel vertical moving plates. The physical configuration is described in figure 4.10 on the following page. The parallel vertical plates are located on the planes $x^* = -L$ and $x^* = L$ and they extend infinitely in the y^* and z^* axes. A constant magnetic field of strength B_0^* is applied across the parallel plates in the positive x^* axis direction. A second material is injected uniformly from the left and there is uniform suction from the right with velocity u_0^* applied at time $t^* = 0$. The equation of continuity for this incompressible fluid is given as $\frac{\partial u^*}{\partial x^*} = 0$ which is integrated to give

$$u^* = u_0^* \tag{4.31}$$

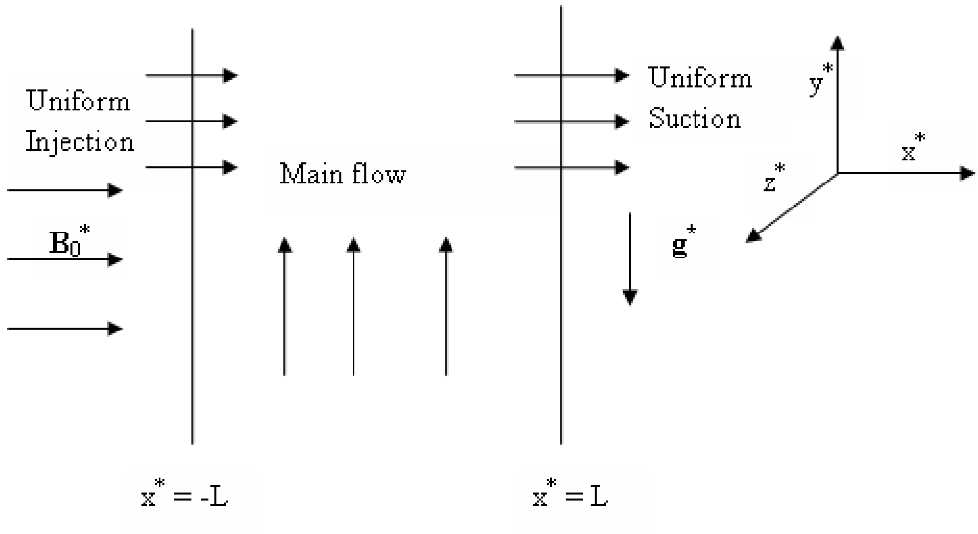

FIGURE 4.10. Fluid flow between moving vertical parallel plates with Hall effect

Existence of the Hall term gives rise to a z^* component of the velocity and in this case the velocity vector of the fluid is given by

$$\boldsymbol{q}^*(x^*, t^*) = u_0^*\boldsymbol{i} + v^*(x^*, t^*)\boldsymbol{j} + w^*(x^*, t^*)\boldsymbol{k} \tag{4.32}$$

Considering a pressure gradient which exists in the y^* direction as a result of the change in elevation and also considering the Boussinesq approximation, and taking the density to depend on temperature and concentration, the body force term takes the form of equation 2.61 on page 30. The component form of the equation of conservation of momentum along the y^* and z^* axes with the electromagnetic force terms takes the form

$$\begin{aligned} \frac{\partial v^*}{\partial t^*} + u_0^*\frac{\partial v^*}{\partial x^*} &= v\frac{\partial^2 v^*}{\partial x^{*2}} + g\left[\beta\left(T^* - T_1^*\right) + \beta_c\left(C^* - C_1^*\right)\right] \\ &+ \frac{\sigma\mu_e^2 H_0}{\rho(1+m^2)}(mw^* - v^*) \end{aligned} \tag{4.33}$$

$$\frac{\partial w^*}{\partial t^*} + u_0^*\frac{\partial w^*}{\partial x^*} = v\frac{\partial^2 w^*}{\partial x^{*2}} - \frac{\sigma\mu_e^2 H_0}{\rho(1+m^2)}(mv^* + w^*) \tag{4.34}$$

When the viscous term and Joule dissipation term are considered, the energy conservation is given by the equation

$$\begin{aligned}\frac{\partial T^*}{\partial t^*}+u_0^*\frac{\partial T^*}{\partial x^*} &= \frac{K}{\rho C_p}\frac{\partial^2 T^*}{\partial x^{*2}}+\frac{\upsilon}{C_p}\left(\left(\frac{\partial v^*}{\partial x^*}\right)^2+\left(\frac{\partial w^*}{\partial x^*}\right)^2\right)\\ &+ \frac{\sigma\mu_e^2 H_0^2}{\rho C_p\,(1+m^2)}(v^{*2}+w^{*2}) \end{aligned} \tag{4.35}$$

The equation of concentration conservation is given as

$$\frac{\partial C^*}{\partial t^*}+u_0^*\frac{\partial C^*}{\partial x^*}=D\frac{\partial^2 C^*}{\partial x^{*2}} \tag{4.36}$$

The two plates are initially stationary but at time $t^*=0$ the plate at the boundary $x^*=L^*$ is set into motion with a velocity U_0 along its plane while the plate at $x^*=-L^*$ is set into motion with the same velocity but in the opposite direction to the plate at $x^*=L^*$. The two plates are considered to be initially isothermal at a temperature T_1^*. At the time t = 0, the temperature of the plate at the boundary $x^*=L^*$ is raised to T_2^* and then kept constant thereafter. The concentration of the fluid at the plates is initially taken as C_1^*. However, at time t = 0, the concentration at the boundary $x^*=L^*$ is increased to C_2^*. Thereafter, the concentrations are maintained at C_1^* and C_2^* at the boundaries $x^*=-L^*$ and $x^*=L^*$ respectively. Considering the no slip condition, the initial conditions for this fluid flow configuration are given as

$$\left.\begin{array}{l} v^*(-L^*,0)=v^*(L^*,0)=0\\ w^*(-L^*,0)=w^*(L^*,0)=0\\ T^*(-L^*,0)=T^*(L^*,0)=T_1^*\\ C^*(-L^*,0)=C^*(L^*,0)=C_1^* \end{array}\right\}\qquad t^*=0 \tag{4.37}$$

The boundary conditions as described above for the fluid flow in consideration are given as

$$\left.\begin{array}{l} v^*(-L^*,t^*)=-U_0\\ w^*(-L^*,t^*)=0\\ v^*(L^*,t^*)=U_0\\ w^*(L^*,t^*)=0\\ T^*(-L^*,t^*)=T_1^*\\ T^*(L^*,t^*)=T_2^*\\ C^*(-L^*,t^*)=C_1^*\\ C^*(L^*,t^*)=C_2^* \end{array}\right\}\qquad t^*>0 \tag{4.38}$$

The non dimensional forms of the equations governing the fluid flow in consideration are

$$\frac{\partial v}{\partial t}+S\frac{\partial v}{\partial x}=\frac{\partial^2 v}{\partial x^2}+Gr\theta+GcC+\frac{M^2}{(1+m^2)}\left[mw-v\right] \tag{4.39}$$

and

$$\frac{\partial w}{\partial t}+S\frac{\partial w}{\partial x}=\frac{\partial^2 w}{\partial x^2}-\frac{M^2}{(1+m^2)}\left[mv+w\right] \tag{4.40}$$

The energy conservation equation is given as

$$\frac{\partial \theta}{\partial t}+S\frac{\partial \theta}{\partial x}=\frac{1}{Pr}\frac{\partial^2 \theta}{\partial x^2}+Ec\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right]+\frac{M^2Ec}{1+m^2}(v^2+w^2) \tag{4.41}$$

The mass conservation equation in non dimensional form is

$$\frac{\partial C}{\partial t}+S\frac{\partial C}{\partial x}=\frac{1}{Sc}\frac{\partial^2 C}{\partial x^2} \tag{4.42}$$

The non dimensional form of the initial and boundary conditions (4.37) and (4.38) are respectively given as

$$\left.\begin{array}{l} v(-L,0)=v(L,0)=0\\ w(-L,0)=w(L,0)=0\\ \theta(-L,0)=\theta(L,0)=0\\ C(-L,0)=C(L,0)=0 \end{array}\right\} \quad t=0 \tag{4.43}$$

$$\left.\begin{array}{l} v(-L,t)=-1\\ w(-L,t)=0\\ v(L,t)=1\\ w(L,t)=0\\ \theta(-L,t)=0\\ \theta(L,t)=1\\ C(-L,t)=0\\ C(L,t)=1 \end{array}\right\} \quad t>0 \tag{4.44}$$

4.2.2 Solution

The solution for the velocities, temperature and concentration, is sought from the system of equations (4.39), (4.40), (4.41) and (4.42) together with the initial conditions (4.43) and the boundary conditions (4.44). Since the system of equations is non linear, we apply the numerical approximation method of finite differences to get the solution to this system of equations. Replacing the derivatives in these equations by finite differences and letting i and j represent x and t respectively at the mesh points we have $x \pm \Delta x = i \pm 1, \quad t \pm \Delta t = j \pm 1$. And the finite difference equations take the form given below.

$$\begin{aligned}\frac{v\left(i,j+1\right)-v(i,j)}{\Delta t} &= \frac{v(i-1,j)-2v(i,j)+v(i+1,j)}{\left(\Delta x\right)^{2}} \\ &- S\left\{\frac{v(i+1,j)-v(i,j)}{\Delta x}\right\} \\ &+ Gr\theta(i,j)+GcC(i,j)+\frac{M^{2}}{1+m^{2}}\left[mw(i,j)-v(i,j)\right]\end{aligned} \tag{4.45}$$

$$\begin{aligned}\frac{w\left(i,j+1\right)-w(i,j)}{\Delta t} &= \frac{w(i-1,j)-2w(i,j)+w(i+1,j)}{\left(\Delta x\right)^{2}} \\ &- S\left\{\frac{w(i+1,j)-w(i,j)}{\Delta x}\right\} \\ &- \frac{M^{2}}{1+m^{2}}\left[mv(i,j)+w(i,j)\right]\end{aligned} \tag{4.46}$$

$$\begin{aligned}\frac{\theta\left(i,j+1\right)-\theta(i,j)}{\Delta t} &= \frac{1}{Pr}\left[\frac{\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)}{(\Delta x)^{2}}\right] \\ &- S\left\{\frac{\theta(i+1,j)-\theta(i,j)}{\Delta x}\right\} \\ &+ Ec\left[\left(\frac{v(i+1,j)-v(i,j)}{\Delta x}\right)^{2}+\left(\frac{w(i+1,j)-w(i,j)}{\Delta x}\right)^{2}\right] \\ &+ \frac{EcM^{2}}{1+m^{2}}\left[v^{2}(i,j)+w^{2}(i,j)\right]\end{aligned} \tag{4.47}$$

$$\begin{aligned}\frac{C(i,j+1)-C(i,j)}{\Delta t} &= \frac{1}{Sc}\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta x)^{2}}\right] \\ &- S\left[\frac{C(i+1,j)-C(i,j)}{\Delta x}\right]\end{aligned} \tag{4.48}$$

The finite difference form of the initial conditions (4.43), and of the boundary conditions (4.44), is given below.

$$\left.\begin{array}{l} v(-40,0)=w(-40,0)=0 \\ v(40,0)=w(40,0)=0 \\ \theta(-40,0)=\theta(40,0)=0 \\ C(-40,0)=C(40,0)=0 \end{array}\right\} \quad j=0 \tag{4.49}$$

$$\left.\begin{array}{l} q(-40,j)=-1 \\ q(40,j)=1 \\ \theta(-40,j)=0 \\ \theta(40,j)=1 \\ C(-40,j)=0 \\ C(40,j)=1 \end{array}\right\} \quad j>0 \tag{4.50}$$

Rearranging the terms of equations (4.45) to (4.48) we can compute consecutive values of the primary velocity v, secondary velocity w, the temperature and the concentration C as shown in equations (4.51) to (4.54) given below.

$$\begin{aligned} v(i,j+1) &= \frac{\Delta t}{(\Delta x)^2}\left[v(i-1,j)-2v(i,j)+v(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta x}\left[v(i+1,j)-v(i,j)\right]+\Delta t\left[\frac{mM^2}{1+m^2}\right]w(i,j) \\ &+ \Delta t\left[Gr\theta(i,j)+GcC(i,j)\right]-\left[\frac{\Delta t M^2}{1+m^2}-1\right]v(i,j) \end{aligned} \tag{4.51}$$

$$\begin{aligned} w(i,j+1) &= \frac{\Delta t}{(\Delta x)^2}\left[w(i-1,j)-2w(i,j)+w(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta x}\left[w(i+1,j)-w(i,j)\right] \\ &- \left[\frac{\Delta t M^2}{1+m^2}-1\right]w(i,j)-\Delta t\left[\frac{mM^2}{1+m^2}\right]v(i,j) \end{aligned} \tag{4.52}$$

$$\begin{aligned} \theta(i,j+1) &= \frac{\Delta t}{\Pr(\Delta x)^2}\left[\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)\right]-S\frac{\Delta t}{\Delta x}\theta(i+1,j) \\ &+ Ec\frac{\Delta t}{(\Delta x)^2}\left[\{v(i+1,j)-v(i,j)\}^2+\{w(i+1,j)-w(i,j)\}^2\right] \\ &+ \left[S\frac{\Delta t}{\Delta x}+1\right]\theta(i,j)+Ec\frac{M^2\Delta t}{1+m^2}\left[v^2(i,j)+w^2(i,j)\right] \end{aligned} \tag{4.53}$$

$$
\begin{aligned}
C(i,j+1) &= \frac{\Delta t}{Sc\,(\Delta x)^2}\left[C(i-1,j)-2C(i,j)+C(i+1,j)\right] \\
&- \frac{S\Delta t}{\Delta x}C(i+1,j)+\left\{\frac{S\Delta t}{\Delta x}+1\right\}C(i,j) \qquad (4.54)
\end{aligned}
$$

4.2.3 Observations

Computations for the primary velocity u, secondary velocity w, temperature θ and concentration C were made for Pr = 0.71, Gr = -0.5 corresponding to heating, Gc =1.5 and $M^2 = 6.0$ The parameters that were varied included the time t, Hall parameter m, suction parameter S, Schmidt number Sc and the Ekcert number Ec. The reference values represented in figures 4.11 on page 96 to figure 4.15 on page 108 by the curve labelled "test"are t = 1, m = 1.0, S = 0.5, Sc = 0.4 and Ec = 0.2 These values of the parameters were varied one at a time and input into a computer program. Computations were done using equations (4.51) to (4.54), the initial conditions (4.49)and the boundary conditions (4.50) and curves plotted for each case. These results were then represented in the figures labelled figure 4.11 on page 96 to figure 4.15 on page 108.

(a)Primary velocity profiles

It is observed that the velocity reduces from that of the left plate to zero, then it is maintained for most of the part of the flow region. As the right plate is approached, the velocity increases up to the velocity of the plate at the right plate. The effects of various parameters on the primary velocity of the fluid flow were considered as discussed below from figure 4.11 on page 96.

Schmidt number

An increase in the value of the Schmidt number Sc, has no effect on the primary velocity near the left plate but it leads to a decrease of the fluid velocity near the right plate. This is because the mass diffusion parameter Sc is directly proportional to the shear stresses and therefore an increase in Sc would result in retarding the fluid motion and hence the velocity.

Hall parameter

As the value of the Hall parameter m increases, primary velocity increases slightly near the right plate. The velocity increases close to the left plate but then soon after it starts decreasing.

Ekcert number

An increase in the value of the Ekcert number Ec leads to a decrease in the primary velocity profiles near the left plate. However as the right plate is approached, the velocity decreases but close to the plate the velocity increases.

Suction

When there is no suction, there is a decrease of the fluid velocity near the right plate but an increase in the velocity near the left plate.

Time

An increase in the time t leads to an increase of the fluid velocity next to both right and left plates. This is because the time is directly proportional to the velocity.

(b) Secondary velocity profiles

Generally, the secondary velocity is positive near the left plate but is reversed near the right plate. The velocity increases from zero at the left plate attains a maximum and then reduces to zero. As the right plate is approached, the velocity increases in the reverse direction attains a maximum then reduces back to zero at the plate.

Schmidt number

An increase in the value of the Schmidt number Sc, has no effect on the secondary velocity near the left plate but it leads to a decrease of the fluid velocity near the right plate.

Hall parameter

An increase in the value of the Hall parameter m leads to a reduction in the secondary velocity near both plates.

Suction

Absence of suction leads to a decrease in the secondary velocity near each of the plates.

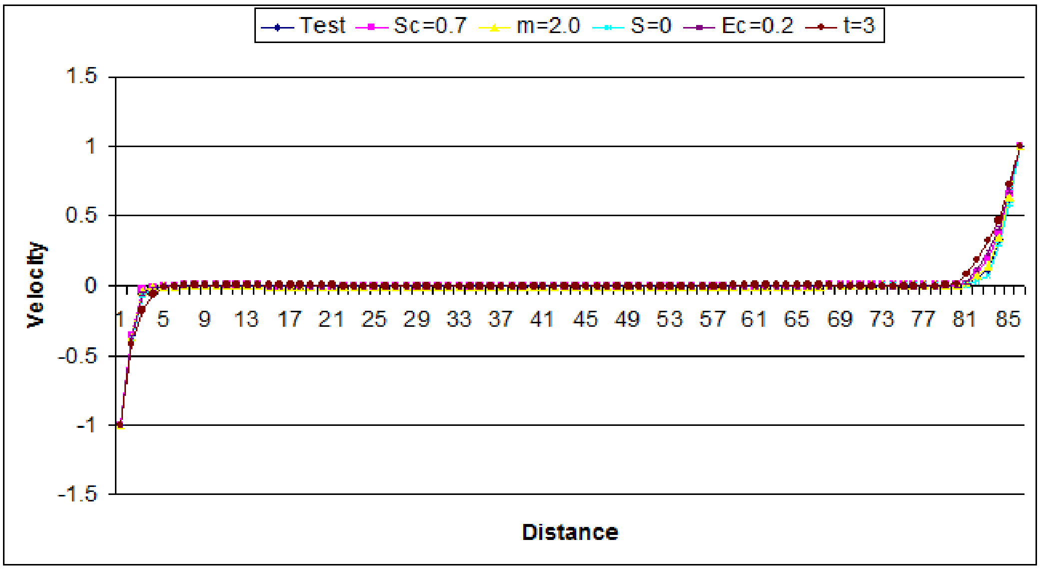

FIGURE 4.11. Primary velocity profiles for vertical plates with both plates moving

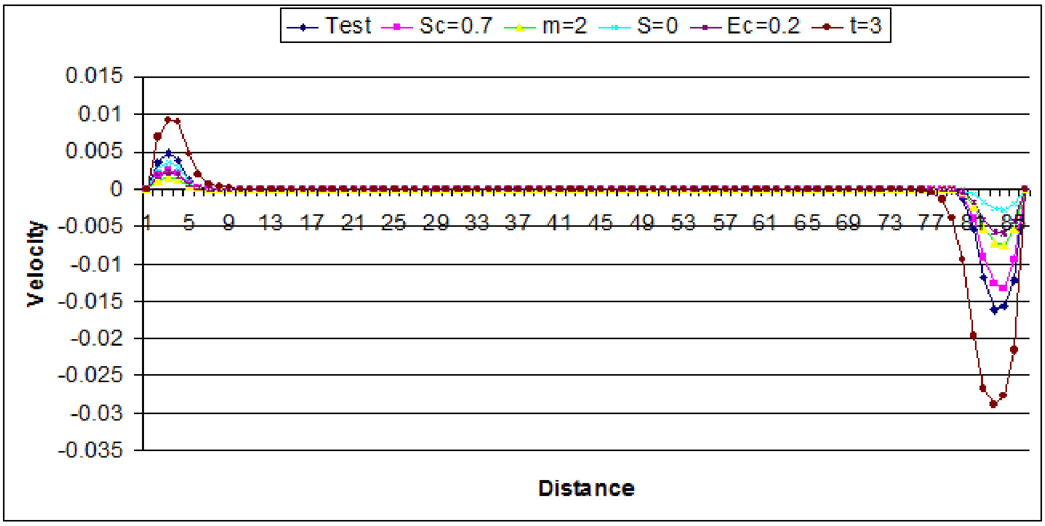

FIGURE 4.12. Secondary velocity profiles for vertical plates with both plates moving

Ekcert number

An increase in the value of the Ekcert number Ec leads to a decrease in the secondary velocity profiles near each of the plates.

Time

An increase in the time t leads to an increase of the secondary fluid velocity next to both right and left plates. This is because the time is directly proportional to the velocity.

(c) Temperature profiles

The effect of the various parameters on the temperature profiles is such that the temperature increases slightly near the left plate, then reduces to zero which is maintained in most of the flow region. As the right plate is approached, the temperature begins to increase up to the velocity of the plate at the right plate.

Hall parameter

There is no significant effect of an increase in the Hall parameter m on the temperature profiles near the right plate. However, there is a decrease in the profiles near the left plate.

Suction

Absence of suction leads to a decrease in the temperature profiles near the right plate and an increase in the profiles near the left plate.

Time

An increase in the time t leads to an increase of the fluid temperature next to both right and left plates. This is because the time is directly proportional to the temperature.

Eckert number

An increase in the Eckert number Ec leads to an increase in the temperature profiles near each of the plates.

(d) Concentration profiles

Generally, the concentration of the fluid increases from zero at the left plate to the concentration of the plate at the right plate. Much of the effect is therefore experienced near the right plate.

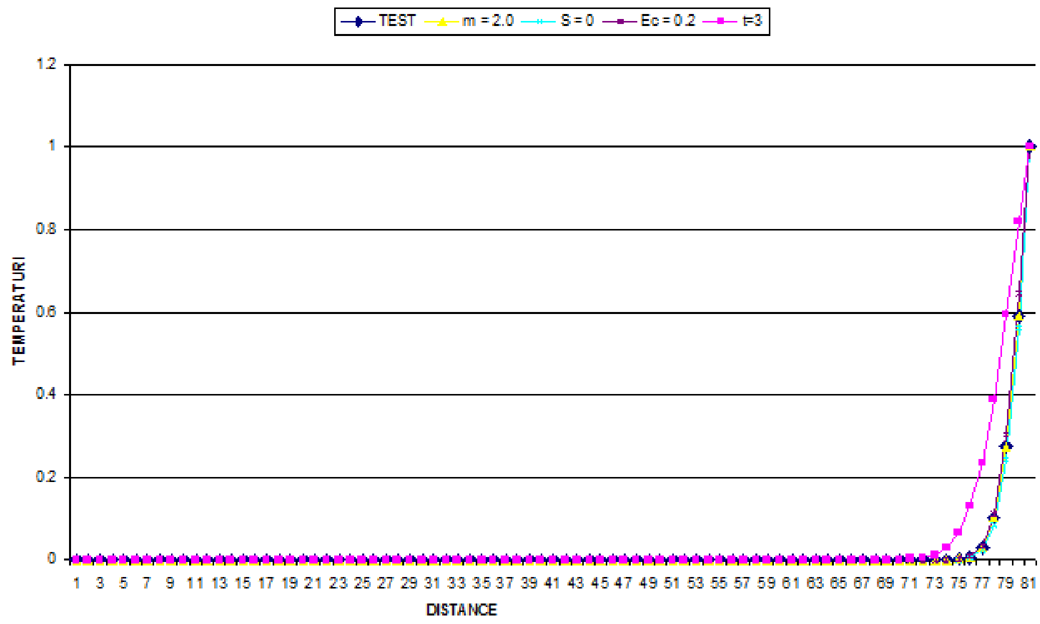

FIGURE 4.13. Temperature profiles for vertical plates with both plates moving

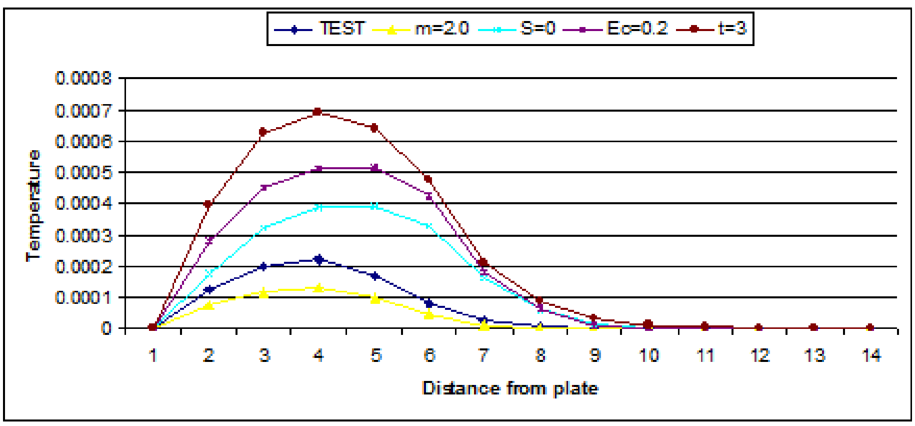

FIGURE 4.14. Temperature profiles near the left plate for vertical plates with both plates moving

Schmidt number

An increase in the Schmidt number Sc, leads to a faster rate of increase of the concentration profiles from the cooler left plate towards the heated right plate. Equivalently it can be stated that the concentration profiles decrease with an increase in the value of the Schmidt number.

Suction

Absence of suction leads to a decrease in the concentration profiles near the right plate.

Eckert number

An increase in the Eckert number Ec leads to an increase in the concentration profiles near the right plate.

Time

An increase in the time t leads to an increase of the fluid concentration near the right plate. This is because the time is directly proportional to the concentration.

4.3 MHD rotating system for a fluid flow between two moving parallel porous plates with effect of Hall current

MHD couette flow in a rotating system permeated by an inclined magnetic field finds wide applications of designing high temperature cooling systems of nuclear reactors, turbine blades and MHD power generators (Ghosh, 2002). In this section, an unsteady MHD fluid is considered to be flowing between two parallel porous plates with heat transfer while putting into consideration the effects of the Hall current. We further consider the two plates to be moving and the whole system is rotated with constant angular velocity. We consider the case where the fluid flows between two horizontal parallel plates.

Consider an incompressible, viscous, heat conducting and electrically conducting fluid flowing between two infinite non conducting porous horizontal plates. The fluid flow is unsteady and a magnetic field is applied perpendicular to the plates. The upper plate is moving with a constant velocity U_0 while the lower plate is moving with a velocity of the same magnitude but in the reverse direction. The induced magnetic field is neglected by assuming a very small magnetic Reynold's number. For this reason, the uniform magnetic field B_0 is considered as the total magnetic field acting on the fluid. The effects of the Hall current, suction and injection are put into consideration while studying the fluid flow in this current chapter. The whole system is then rotated with a constant angular velocity Ω.

4.3.1 Mathematical formulation

Let the two electrically non conducting horizontal plates be located on the planes $y = \pm h$. The plates are of infinite length in the x and z directions i.e. $-\infty < x < \infty$ and $-\infty < z < \infty$. We consider the fluid to be flowing between the two plates under the influence of a constant pressure gradient $\frac{dP}{dx}$ in the x direction, and a uniform injection from below and suction from above with a constant velocity v_0. The pressure gradient is applied at time t = 0. The upper and lower plates are initially at rest but are set into motion at time t = 0. A uniform magnetic field B_0 acts on the whole system in the positive y direction as shown in figure 4.1 on page 70. However for this section the whole system is rotated with a constant angular velocity Ω about the y axis. For the fluid flow being considered, all quantities depend on the space coordinate y and time t except the pressure gradient $\frac{dP}{dx}$ which is assumed constant. The velocity of the fluid is given as $\mathbf{q}(y,t) = u(y,t)\mathbf{i} + v_0\mathbf{j} + w(y,t)\mathbf{k}$ and the applied magnetic field acts along the y axis and is given by $\boldsymbol{B}_0 = \langle 0, B_0, 0\rangle$. When the strength of the magnetic field is large, Ohm's law is modified to include the Hall currents. The horizontal plates are considered to be moving in opposite directions relative to each other with a velocity U_0 of equal magnitude.

4.3.2 Governing equations in dimensional and non dimensional form

When the effect of rotation is considered, the coriolis force has to be included in the momentum equation. Considering a rotating frame of reference with a uniform angular velocity Ω along the y axis, the coriolis force will be given by $2\Omega\boldsymbol{n} \times \mathbf{q}$. Here $\mathbf{n}$ is a unit vector along the y axis. The equations of motion considered in equation 4.1 and equation 4.2 on page 71 are then modified to become

$$\frac{\partial u^*}{\partial t^*} + v_0^*\frac{\partial u^*}{\partial y^*} + 2\Omega w^* = -\frac{1}{\rho}\frac{dP}{dx} + v\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(u^* + mw^*) \tag{4.55}$$

$$\frac{\partial w^*}{\partial t^*} + v_0^*\frac{\partial w^*}{\partial y^*} - 2\Omega u^* = v\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mu^* - w^*) \tag{4.56}$$

Dividing each of the terms in equation (4.55) and equation (4.56) by $\frac{U_0^3}{v}$ we have

$$\begin{aligned} \frac{v}{U_0^3} \times \left(\frac{\partial u^*}{\partial t^*} + v_0^*\frac{\partial u^*}{\partial y^*} + 2\Omega w^*\right) &= -\frac{dP^*}{dx^*} \times \frac{v}{\rho U_0^3} + \frac{\mu v}{U_0^3} \times \frac{\partial^2 u^*}{\partial y^{*2}} \\ &- \frac{v}{U_0^3} \times \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(u^* + mw^*) \end{aligned}$$

$$\frac{v}{U_0^3} \times \left(\frac{\partial w^*}{\partial t^*} + v_0^*\frac{\partial w^*}{\partial y^*} - 2\Omega u^*\right) = \frac{v}{U_0^3} \times v\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{v}{U_0^3} \times \frac{\sigma\mu_e^2 H_0^2}{\rho(1+m^2)}(mu^* - w^*)$$

Consider the rotation parameter $Er = \frac{\Omega v}{U_o^2}$. On non dimensionalization of the terms in equation (4.55) and equation (4.56) we have the momentum equations given as

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} + 2wEr = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2}(u + mw) \tag{4.57}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} - 2uEr = \frac{\partial^2 w}{\partial y^2} + \frac{M^2}{1+m^2}(mu - w) \tag{4.58}$$

The energy conservation equation and the concentration equation are given respectively as equation 4.59 and equation 4.60 which are similar to the equations 4.3 and 4.4 on page 71 considered in chapter four.

$$\begin{aligned} \rho C_p\left(\frac{\partial T^*}{\partial t^*} + v_0^*\frac{\partial T^*}{\partial y^*}\right) &= K\frac{\partial^2 T^*}{\partial y^{*2}} + \mu\left(\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \left(\frac{\partial w^*}{\partial y^*}\right)^2\right) \\ &+ \frac{\sigma\mu_e^2 H_0^2}{1+m^2}(u^{*2} + w^{*2}) \end{aligned} \tag{4.59}$$

$$\frac{\partial C^*}{\partial t^*} + v_0^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \tag{4.60}$$

For the fluid flow in consideration, let the two plates be initially isothermal at temperature T_1^*. The temperature of the upper plate is then raised to T_2^* and thereafter the lower and upper plates are maintained at two different but constant temperatures T_1^* and T_2^* respectively. The plates are initially at rest but at time t = 0 the upper plate is set into motion with a velocity U_0 and the lower plate is also set into motion with the same velocity but in the opposite direction. Let the concentration at both plates be C_1^* initially. The concentration at the upper plate is then increased to C_2^* at time t = 0 and the concentrations at each of the boundaries are kept constant thereafter. From the definition of the fluid flow problem under consideration, the initial and boundary conditions are respectively given below as

$$\left.\begin{array}{l} u^*(-h^*,t^*) = w^*(-h^*,t^*) = 0 \\ u^*(h^*,t^*) = w^*(h^*,t^*) = 0 \\ T^*(-h^*,t^*) = T^*(h^*,t^*) = T_1^* \\ C^*(-h^*,t^*) = C^*(h^*,t^*) = C_1^* \end{array}\right\} \quad t^* = 0 \tag{4.61}$$

$$\left.\begin{array}{l} u^*(-h^*,t^*) = -U_0 \\ w^*(-h^*,t^*) = 0 \\ u^*(h^*,t^*) = U_0 \\ w^*(h^*,t^*) = 0 \\ T^*(-h^*,t^*) = T_1^* \\ T^*(h^*,t^*) = T_2^* \\ C^*(-h^*,t^*) = C_1^* \\ C^*(h^*,t^*) = C_2^* \end{array}\right\} \quad t^* > 0 \tag{4.62}$$

The non dimensional form of the momentum conservation equations, the energy equation and the mass conservation equation is given from equations (4.57), (4.58), (4.59) and (4.60) above as equations (4.63) to equation (4.66) below.

$$\frac{\partial u}{\partial t} + S\frac{\partial u}{\partial y} + 2wEr = -\frac{dP}{dx} + \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+m^2}(u + mw) \tag{4.63}$$

$$\frac{\partial w}{\partial t} + S\frac{\partial w}{\partial y} - 2uEr = \frac{\partial^2 w}{\partial y^2} + \frac{M^2}{1+m^2}(mu - w) \tag{4.64}$$

$$\frac{\partial \theta}{\partial t} + S\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\frac{\partial \boldsymbol{q}}{\partial y}\cdot\frac{\partial \bar{\boldsymbol{q}}}{\partial y} + \frac{EcM^2}{1+m^2}\mathbf{q}\bar{\mathbf{q}} \tag{4.65}$$

$$\frac{\partial C}{\partial t}+S\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{4.66}$$

The non dimensional forms of the initial conditions (4.61) are given as equations (4.67) and the boundary conditions in the equation (4.62) are given in non dimensional form as equations (4.68).

$$\left.\begin{array}{l} q(-L,0)=q(L,0)=0\\ \theta(-L,0)=\theta(L,0)=0\\ C(-L,0)=C(L,0)=0 \end{array}\right\} \qquad t=0 \tag{4.67}$$

$$\left.\begin{array}{l} q(-L,t)=-1\\ q(L,t)=1\\ \theta(-L,t)=0\\ \theta(L,t)=1\\ C(-L,t)=0\\ C(L,t)=1 \end{array}\right\} \qquad t>0 \tag{4.68}$$

4.3.3 Solution

We seek solution of the system of equations (4.63),(4.64),(4.65) and (4.66) together with the initial conditions (4.67) and the boundary conditions (4.68). The system of equations is non linear and we apply the numerical approximation method of finite differences in the solution. Using forward differences, the finite difference form of equations governing the fluid flow under consideration i.e. equations (4.63),(4.64),(4.65) and (4.66) have the form

$$\begin{aligned} \frac{u\,(i,j+1)-u(i,j)}{\Delta t} &= -\frac{dP}{dx}+\frac{u(i-1,j)-2u(i,j)+u(i+1,j)}{(\Delta y)^2}\\ &- S\left\{\frac{u(i+1,j)-u(i,j)}{\Delta y}\right\}\\ &- 2Erw(i,j)-\frac{M^2}{1+m^2}\left[u(i,j)+mw(i,j)\right] \end{aligned} \tag{4.69}$$

$$\begin{aligned} \frac{w\,(i,j+1)-w(i,j)}{\Delta t} &= \frac{w(i-1,j)-2w(i,j)+w(i+1,j)}{(\Delta y)^2}\\ &- S\left\{\frac{w(i+1,j)-w(i,j)}{\Delta y}\right\}\\ &+ 2Eru(i,j)\frac{M^2}{1+m^2}\left[mu(i,j)-w(i,j)\right] \end{aligned} \tag{4.70}$$

$$\begin{aligned}\frac{\theta\,(i,j+1)-\theta(i,j)}{\Delta t} &= \frac{1}{Pr}\left[\frac{\theta(i-1,j)-2\theta(i,j)+\theta(i+1,j)}{(\Delta y)^2}\right]\\ &- S\left\{\frac{\theta(i+1,j)-\theta(i,j)}{\Delta y}\right\}\\ &+ Ec\left[\left(\frac{u(i+1,j)-u(i,j)}{\Delta y}\right)^2+\left(\frac{w(i+1,j)-w(i,j)}{\Delta y}\right)^2\right]\\ &+ \frac{EcM^2}{1+m^2}\left[u(i,j)^2+w(i,j)^2\right] \qquad (4.71)\end{aligned}$$

$$\begin{aligned}\frac{C(i,j+1)-C(i,j)}{\Delta t} &= \frac{1}{Sc}\left[\frac{C(i-1,j)-2C(i,j)+C(i+1,j)}{(\Delta y)^2}\right]\\ &- S\left[\frac{C(i+1,j)-C(i,j)}{\Delta y}\right] \qquad (4.72)\end{aligned}$$

The finite difference form of the initial conditions (4.67) and the boundary conditions (4.68) is given below.

$$\left.\begin{array}{l} q(-40,0)=q(40,0)=0\\ \theta(-40,0)=\theta(40,0)=0\\ C(-40,0)=C(40,0)=0\end{array}\right\}\quad j=0 \qquad (4.73)$$

$$\left.\begin{array}{l} q(-40,j)=-1\\ q(40,j)=1\\ \theta(-40,j)=0\\ \theta(40,j)=1\\ C(-40,j)=0\\ C(40,j)=1\end{array}\right\}\quad j>0 \qquad (4.74)$$

In this case i and j represent the space coordinate y and the time coordinate t respectively. Rearranging each of these equations enables us to compute consecutive terms of the primary velocity u, the secondary velocity w, the temperature θ and the concentration C using the initial values and boundary conditions given in equations (4.73) and (4.74). Rearrangement of the equations (4.69) to (4.72) gives equations (4.75) to (4.78) below. The solutions to these equations together with the initial conditions (4.67) and boundary conditions (4.68) are computed using an appropriate computer program.

$$\begin{aligned} u(i,j+1) &= -\Delta t\frac{dP}{dx} + \frac{\Delta t}{(\Delta y)^2}\left[u(i-1,j) - 2u(i,j) + u(i+1,j)\right] \\ &- 2Er\Delta t w(i,j) - \frac{S\Delta t}{\Delta y}\left[u(i+1,j) - u(i,j)\right] \\ &- \left[\frac{\Delta t M^2}{1+m^2} - 1\right] u(i,j) - \Delta t\left[\frac{mM^2}{1+m^2}\right] w(i,j) \end{aligned} \tag{4.75}$$

$$\begin{aligned} w(i,j+1) &= \frac{\Delta t}{(\Delta y)^2}\left[w(i-1,j) - 2w(i,j) + w(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta y}\left[w(i+1,j) - w(i,j)\right] + \Delta t\left[\frac{mM^2}{1+m^2}\right] u(i,j) \\ &+ 2Er\Delta t w(i,j) - \left[\frac{\Delta t M^2}{1+m^2} - 1\right] w(i,j) \end{aligned} \tag{4.76}$$

$$\begin{aligned} \theta(i,j+1) &= \frac{\Delta t}{\Pr(\Delta y)^2}\left[\theta(i-1,j) - 2\theta(i,j) + \theta(i+1,j)\right] - S\frac{\Delta t}{\Delta y}\theta(i+1,j) \\ &+ Ec\frac{\Delta t}{(\Delta y)^2}\left[\{u(i+1,j) - u(i,j\}^2 + \{w(i+1,j) - w(i,j\}^2\right] \\ &+ Ec\frac{M^2\Delta t}{1+m^2}\left[u(i,j)^2 + w(i,j)^2\right] + \left[S\frac{\Delta t}{\Delta y} + 1\right]\theta(i,j) \end{aligned} \tag{4.77}$$

$$\begin{aligned} C(i,j+1) &= \frac{\Delta t}{Sc(\Delta y)^2}\left[C(i-1,j) - 2C(i,j) + C(i+1,j)\right] \\ &- \frac{S\Delta t}{\Delta y}C(i+1,j) \\ &+ \left\{\frac{S\Delta t}{\Delta y} + 1\right\} C(i,j) \end{aligned} \tag{4.78}$$

4.3.4 Calculation of the rate of mass transfer, skin friction and the rate of heat transfer

The equations derived in section 4.1.4 on page 75 are used to obtain values of the skin friction, the rate of heat transfer as well as the rate of mass transfer on the flow field for the rotation parameter Er , Schmidt number Sc, Hall parameter m, Suction parameter S, Eckert number Ec, Pressure gradient $\frac{dP}{dx}$ and time t. The equations used are therefore stated below as equations (4.79) to (4.86).

The skin friction on the lower and upper plates in the direction of the x axis is obtained respectively from equations 4.79 and 4.80 given below.

$$\tau_x = -(\mathrm{b}_1 - 2\mathrm{d}_1\mathrm{l} + \mathrm{f}_1\mathrm{t}) \tag{4.79}$$

$$\tau_x = -(\mathrm{b}_1 + 2\mathrm{d}_1\mathrm{l} + \mathrm{f}_1\mathrm{t}) \tag{4.80}$$

Similarly, the skin friction on the lower and upper plates in the direction of the z axis is obtained from equations 4.81 and 4.82 given below.

$$\tau_z = -(\mathrm{b}_2 - 2\mathrm{d}_2\mathrm{l} + \mathrm{f}_2\mathrm{t}) \tag{4.81}$$

$$\tau_z = -(\mathrm{b}_2 + 2\mathrm{d}_2\mathrm{l} + \mathrm{f}_2\mathrm{t}) \tag{4.82}$$

The rate of heat transfer on the lower and upper plates is obtained from equations 4.83 and 4.84 below.

$$Nu = -(\mathrm{b}_3 - 2\mathrm{d}_3\mathrm{l} + \mathrm{f}_3\mathrm{t}) \tag{4.83}$$

$$Nu = -(\mathrm{b}_3 + 2\mathrm{d}_3\mathrm{l} + \mathrm{f}_3\mathrm{t}) \tag{4.84}$$

The rate of mass transfer on the lower and upper plates is calculated respectively from equations 4.85 and 4.86 below.

$$Sh = -(\mathrm{b}_4 - 2\mathrm{d}_4\mathrm{l} + \mathrm{f}_4\mathrm{t}) \tag{4.85}$$

$$Sh = -(\mathrm{b}_4 + 2\mathrm{d}_4\mathrm{l} + \mathrm{f}_4\mathrm{t}) \tag{4.86}$$

4.3.5 Observations and discussions

Computations for the primary velocity u, secondary velocity w, temperature θ and concentration C were made for various parameters with Pr = 0.71 and $M^2 = 6.0$. The parameters that were varied included the pressure gradient $\frac{dP}{dx}$, time t, Hall parameter m, suction parameter S, Schmidt number Sc, rotation parameter Er and the Ekcert number Ec. The reference values represented in figures 4.16 to 4.23 by the curve labeled "test"are $\frac{dP}{dx}$ = 5.0, m = 1.0, S = 0.5, Sc = 0.4, Er = 0.05 and Ec = 0.2. These values of the parameters were varied one at a time and input into a computer program. Computations were done using equations (4.75) to (4.78), the initial conditions (4.67) and the boundary conditions (4.68) and curves were plotted for each case. These results were then represented in the figures labelled figure 4.16 to figure 4.23.

The values of velocities, temperature, and concentration obtained in section 4.3.3 are used for different values of the rotation parameter Er, Schmidt number Sc, Hall parameter m, Suction parameter S, Eckert number Ec, Pressure gradient $\frac{dP}{dx}$ and time t are used to obtain the values of the skin friction and the rates of heat and mass transfer. The Tables represent the variation in the rates of heat and mass transfer as well as the skin friction on the laminar thermal, concentration and velocity boundary layers respectively. The reference values used for the parameters are Er=0.05 for the rotation parameter , Sc=0.4 for the Schmidt number , m=1.0 for the Hall parameter, S= -0.5 for the Suction parameter, Ec=0.2 for the Eckert number, $\frac{dP}{dx}$ = 5.0, for the pressure gradient and t=0.25 for the time. Each of the parameters is varied and the results are presented in tables 4.5 on page 114 to table 4.8 on page 117. The observations made from these tables are then discussed after the concentration profiles.

a) Primary velocity profiles

We discuss how each of the parameters affects the primary velocity profiles of the fluid flow as represented by the graphs in figure 4.16 on the next page. For clarity, the profiles near the lower plate are shown in figure 4.17 on page 109 while the profiles near the upper plate are shown in figure 4.18 on page 109.

Pressure gradient

From figures 4.16, 4.17 and 4.18 it is observed that very close to the lower plate, both a positive and negative pressure gradient leads to a reduced fluid flow velocity until a free stream velocity is reached. However, a negative pressure gradient aids the fluid flow for a small region before the free stream velocity is

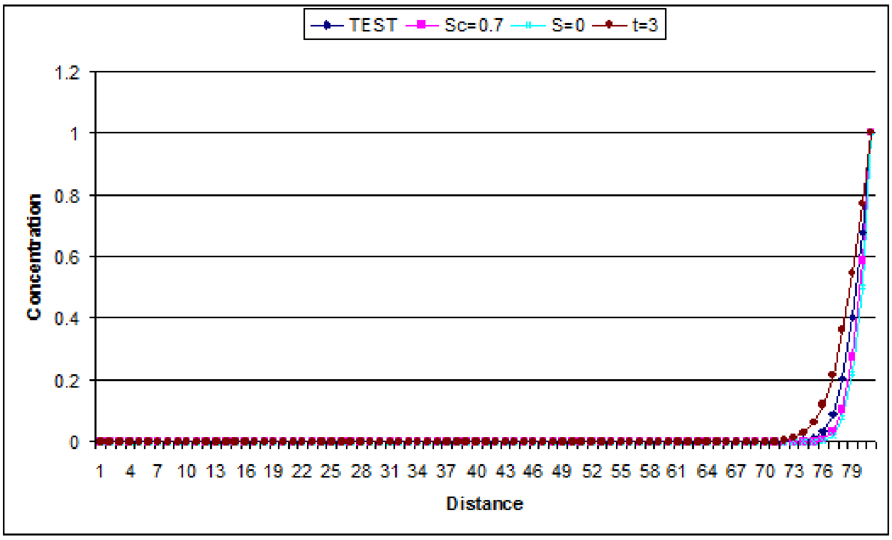

FIGURE 4.15. Concentration profiles for vertical plates with both plates moving

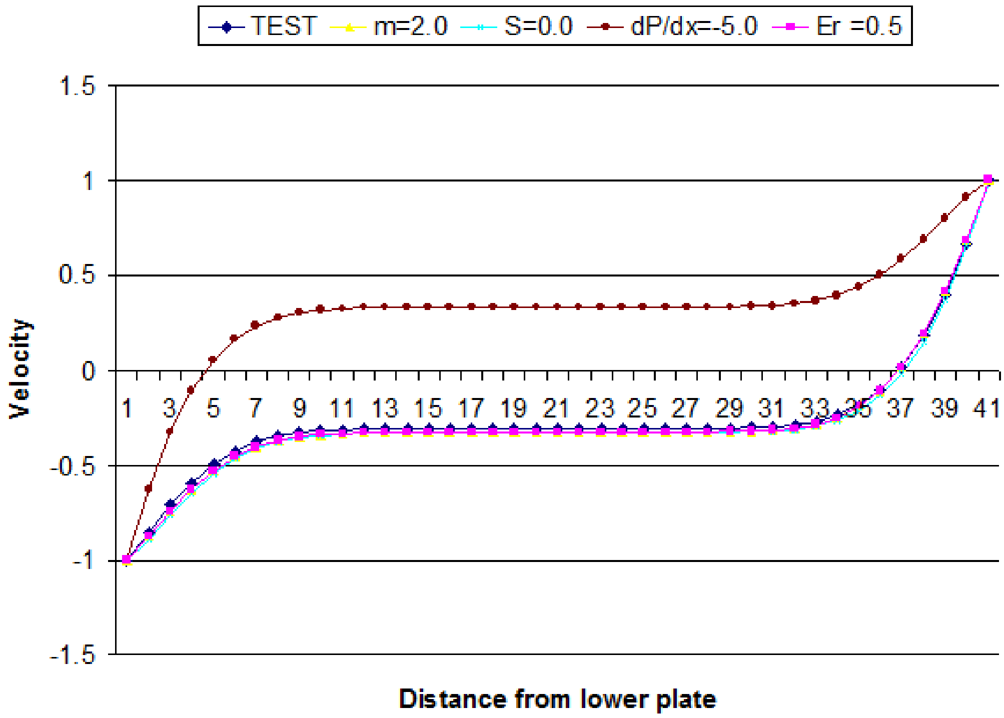

FIGURE 4.16. Primary velocity profiles in a rotating system for horizontal plates with both plates moving

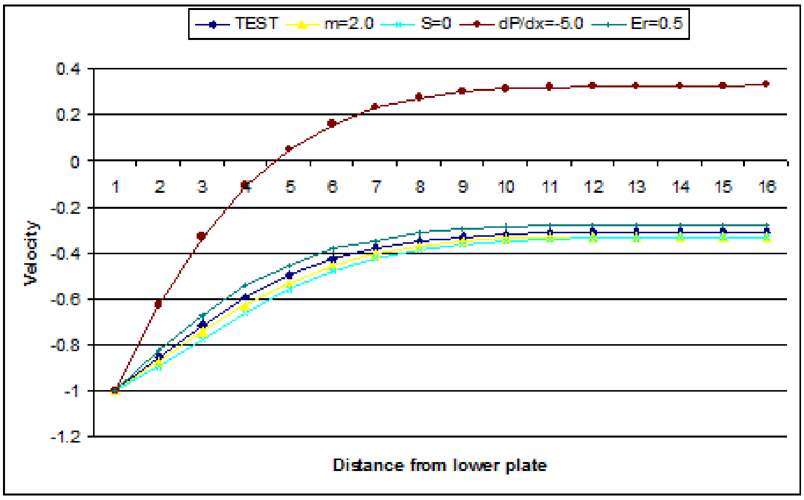

FIGURE 4.17. Primary velocity profiles near the lower plate in a rotating system for horizontal plates with both plates moving

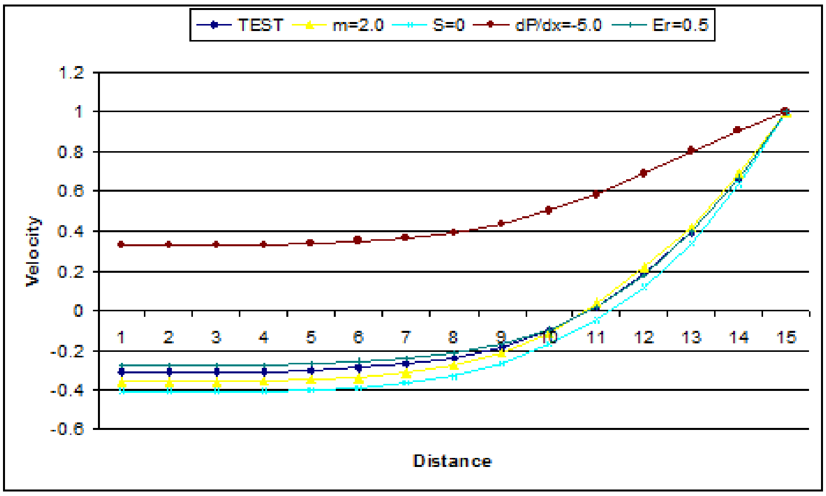

FIGURE 4.18. Primary velocity profiles near the upper plate in a rotating system for horizontal plates with both plates moving

achieved. This means that the inertia force dominates the fluid flow close to the lower plate. For a negative pressure gradient, the velocity increases further from the free stream velocity to the velocity of the upper plate. It is observed that close to the lower boundary, the primary velocity decreases but then soon after the velocity is maintained at a constant level except for the negative pressure gradient as discussed above. As the upper boundary is approached, the velocity in the case of positive pressure gradient reduces up to a point when u = 0 (stationary point) and then it increases suddenly until it reaches the velocity of the plate.

Suction

When there is no suction, the fluid velocity reduces in a lesser degree in the negative direction. The velocity starts reducing from the free stream velocity up to stagnation point and then increases up to the velocity of the upper plate at a faster rate as observed in figure 4.18 on the preceding page. The velocity profiles are thus reduced near the upper plate.

Hall parameter

As the value of the Hall parameter m increases, figure 4.16 on page 108 shows that the reduction in the velocity is less and the free stream velocity is higher in the negative direction. From figure 4.18 on the preceding page it is observed that as the upper plate is approached after stagnation velocity is reached, the primary velocity is more for increased value of the Hall parameter. This is because the effective conductivity decreases with increase in the Hall parameter which then reduces the magnetic dumping force on the velocity causing the velocity to increase.

Rotation parameter

As the value of the rotation parameter Er increases, figure 4.16 on page 108 shows that the reduction in the velocity is more. From figure 4.18 on the preceding page it is observed that as the upper plate is approached after stagnation velocity is reached, the primary velocity is less for increased value of the rotation parameter.

b) Secondary velocity profiles

We discuss how each of the parameters affects the secondary velocity profiles of the fluid flow as observed in figure 4.19 on page 114. It is observed that there is a flow in the negative direction near the lower plate for all cases. With the exception of the case of negative pressure gradient, the velocity is sustained in the reversed direction for the larger part of the flow region. The velocity then reduces, reaches a stagnation point and then increases in the positive direction up to a maximum and then reduces to zero at the upper plate.

Pressure gradient

Close to the lower plate, both a negative and positive pressure gradient leads to the fluid flowing in the negative direction. For a positive pressure gradient, a maximum is attained and then the velocity reduces to a constant velocity in the negative direction. The velocity then reduces further as the upper plate is approached, reaches a stagnation point and then increases to a maximum in the positive direction and then reduces to zero at the upper plate. For a negative pressure gradient, the velocity increases in the negative direction near the lower plate but soon after reduces to zero then increases to a lesser maximum in the positive direction. This is maintained for some time but as the upper plate is approached the velocity increases further to a higher maximum and then reduces to zero at the plate.

Suction

Absence of suction leads to a similar behavior as the case for a positive pressure gradient. However the maximum attained in the absence of suction is lower than the case when suction is considered to exist.

Hall parameter

An increase in the magnitude of the Hall parameter leads to a similar behavior as the case for a positive pressure gradient. However the maximum attained with an increase in the Hall parameter is lower than the case when a smaller value of the Hall parameter is considered.

Rotation parameter

An increase in the magnitude of the rotation parameter leads to a similar behavior as the case for a positive pressure gradient. The maximum attained with an increase in the rotation parameter is higher than the case when a smaller value of the rotation parameter is considered.

c) Temperature profiles

A change in each of the parameters leads to a slight increase of the temperature profiles near the lower plate. This temperature then reduces to zero and then increases to the temperature of the upper plate as the upper plate is approached. Significant effects are observed as you get closer to the upper plate. Considering figure 4.20, we discuss each of the parameters. To clearly see the profiles near the lower and upper plates consider respectively the figure 4.21 on page 115 and figure 4.22 on page 116.

Pressure gradient

When the imposed pressure gradient is negative, the temperature increases slightly near the lower plate then reduces to zero as seen in figure 4.21 on page 115. The change is however not significant near the upper plate.

Hall parameter

An increase in the Hall parameter m leads to a lesser increase in the temperature profiles near the lower plate. There is a slight decrease in the temperature profiles near the upper plate. This is because an increase in the Hall parameter leads to a decrease in the Joule dissipation.

Suction

Absence of suction leads to the temperature increasing at a lesser extent near the lower plate. The rate of increase is more near the upper plate as observed from figure 4.22 on page 116. The temperature profiles are however reduced near the upper plate.

Rotation parameter

A reduction in the value of the rotation parameter Er leads to reduced temperature profiles near both the lower and upper plates.

d) Concentration profiles

From figure 4.23, it is observed that there is little effect produced by the change in the parameters near the lower plate but the effects are observed as you approach the upper plate.

Schmidt number

An increase in the value of the Schmidt number Sc causes the concentration to start increasing closer to the upper plate. The concentration profiles are thus reduced near the upper plate. This is because Sc is inversely proportional to the concentration and therefore an increase in Sc leads to a decrease in the concentration profiles.

Suction

Removal of suction leads to the concentration profiles being reduced near the upper plate.

e) Skin friction τ_x along the plates

The skin friction τ_x along the lower plate is positive while that along the upper plate is negative. We discuss the effect of changing each of the parameters on the skin friction τ_x with reference to the table 4.5 on the following page.

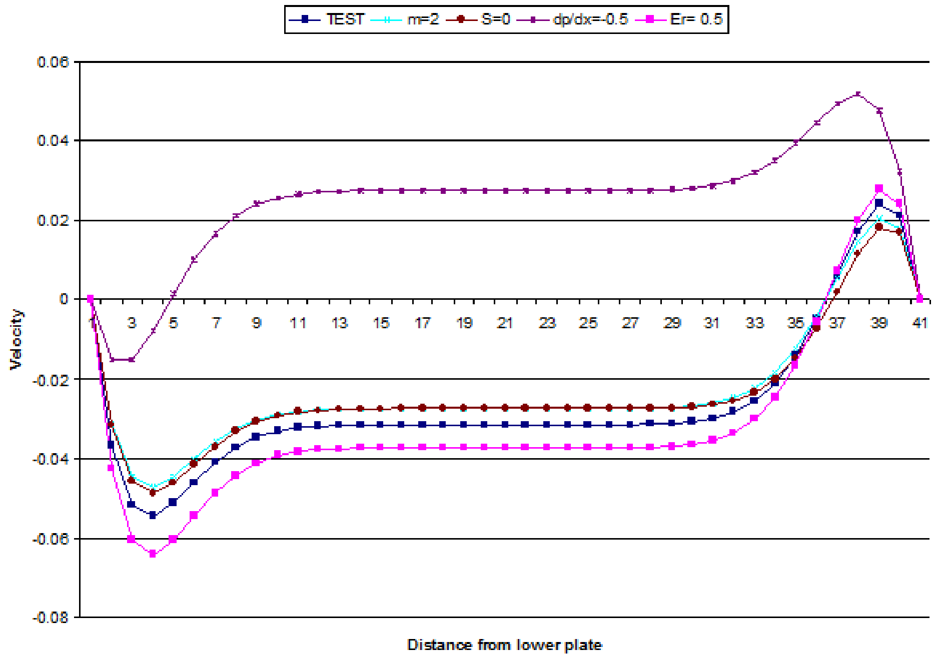

FIGURE 4.19. Secondary velocity profiles in a rotating system for horizontal plates with both plates moving

							lower plate	upper plate
	m	S	Ec	$\frac{dP}{dx}$	Er	t	τ_x	τ_x
Test	1	-0.5	0.2	5	0.05	0.25	261.3016759	-270.8250345
2	2	-0.5	0.2	5	0.05	0.25	260.2973563	-269.8118537
3	2	0	0.2	5	0.05	0.25	258.5962882	-268.1016467
5	2	-0.5	0.02	-5	0.05	0.25	432.2197712	-443.8879626
6	2	-0.5	0.02	5	0.5	0.25	260.531824	-270.0495939
7	1	-0.5	0.2	5	0.05	0.45	474.15236	-483.6557186

TABLE 4.5. Skin friction along the plates for a fluid flow between two horizontal parallel plates in a rotating system

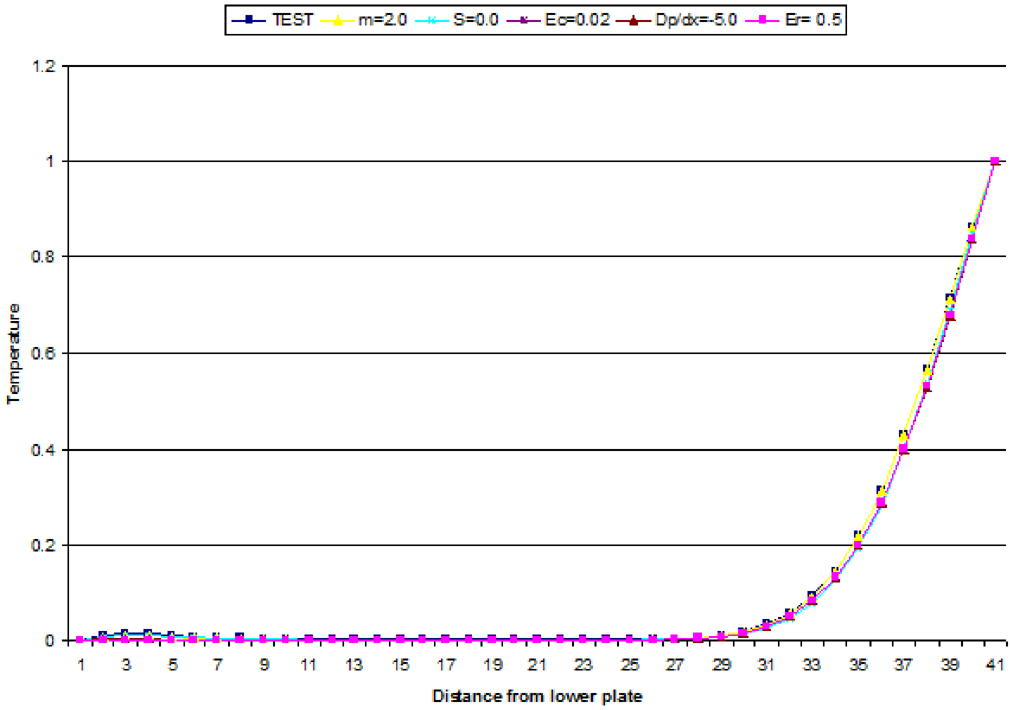

FIGURE 4.20. Temperature profiles in a rotating system for horizontal plates with both plates moving

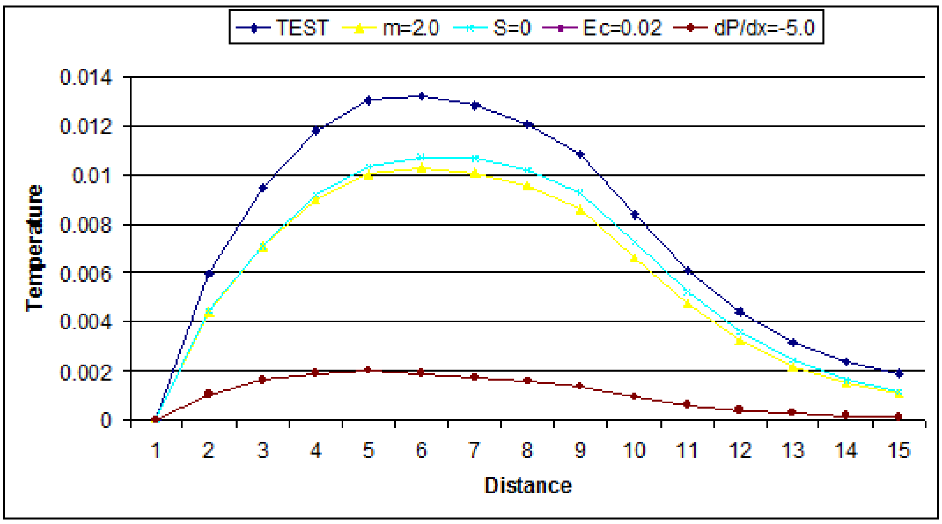

FIGURE 4.21. Temperature profiles near the lower plate in a rotating system for horizontal plates with both plates moving

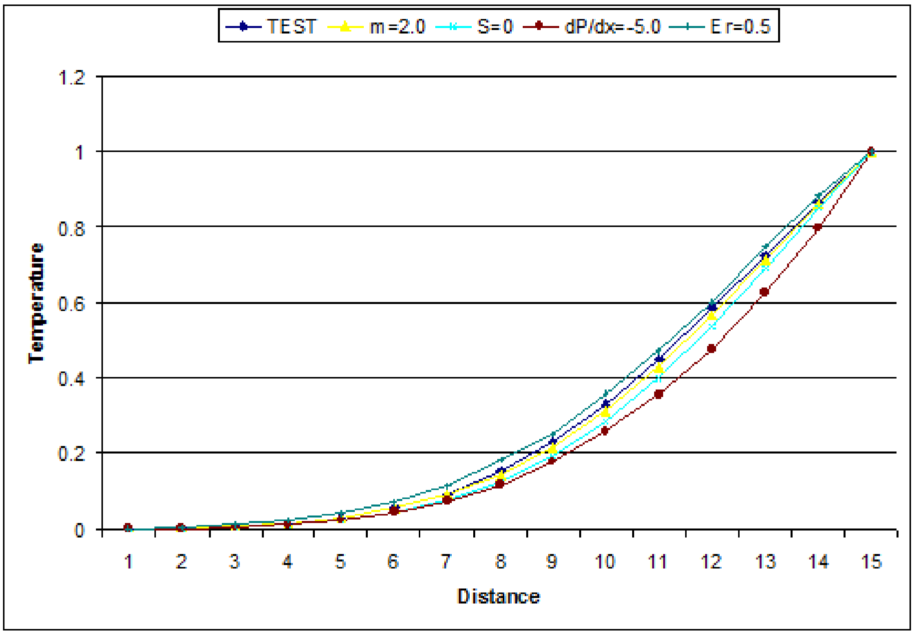

FIGURE 4.22. Temperature profiles near the upper plate in a rotating system for horizontal plates with both plates moving

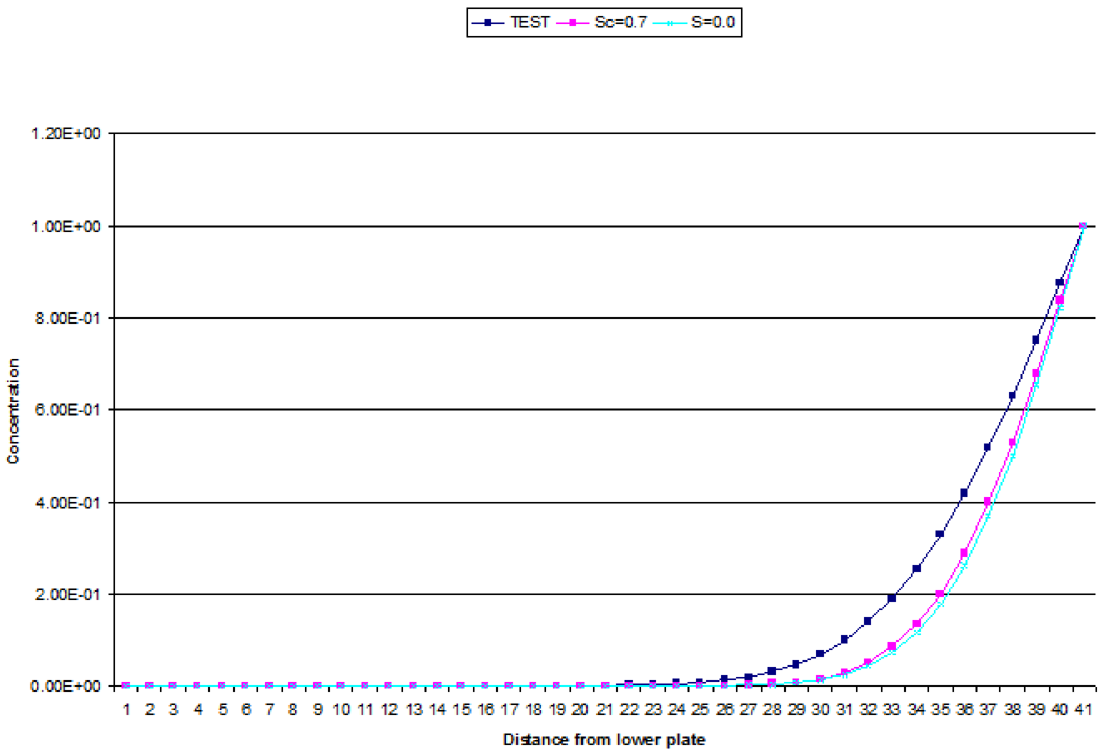

FIGURE 4.23. Concentration profiles in a rotating system for horizontal plates with both plates moving

							lower plate	upper plate
	m	S	Ec	$\frac{dP}{dx}$	Er	t	τ_z	τ_z
Test	1	-0.5	0.2	5	0.05	0.25	-3.321048761	0.478593263
2	2	-0.5	0.2	5	0.05	0.25	-3.597517995	0.75806557
3	2	0	0.2	5	0.05	0.25	-3.697778864	0.866512421
5	2	-0.5	0.02	-5	0.05	0.25	0.807476216	-3.781148791
6	2	-0.5	0.02	5	0.5	0.25	-3.22895711	0.394285985
7	1	-0.5	0.2	5	0.05	0.45	-4.84090557	1.998450072

TABLE 4.6. Skin friction perpendicular to two horizontal parallel plates for a rotating system

							lower plate	upper plate
	m	S	Ec	$\frac{dP}{dx}$	Er	t	Nu	Nu
Test	1	-0.5	0.2	5	0.05	0.25	-5.642631146	2.807074714
2	2	-0.5	0.2	5	0.05	0.25	-5.368511965	2.526530436
3	2	0	0.2	5	0.05	0.25	-5.220682905	2.37228143
4	2	-0.5	0.02	5	0.05	0.25	-4.722813231	1.872199038
5	2	-0.5	0.02	-5	0.05	0.25	-4.879271701	2.03295179
6	2	-0.5	0.02	5	0.5	0.25	-4.723015038	1.872410216
7	1	-0.5	0.2	5	0.05	0.45	-9.022513491	6.186957058

TABLE 4.7. Rates of heat transfer for a fluid flow between two parallel horizontal plates in a rotating system

			lower plate	upper plate
	Sc	t	Sh	Sh
Test	0.4	0.25	-4.651773008	1.800199631
1	0.7	0.25	-4.651773001	1.800199623
7	0.4	0.45	-7.232562064	4.380988686

TABLE 4.8. Rates of mass transfer for a fluid flow between two parallel horizontal plates in a rotating system

Hall parameter m An increase in the value of the hall parameter $m, m = 2.0$ leads to a decrease in the magnitude of the skin friction τ_x along both the lower and upper plates.

Suction parameter S Removal of suction $S = 0$ leads to a decrease in the magnitude of the skin friction τ_x along both the lower and upper plates.

Pressure gradient A negative pressure gradient $\frac{dP}{dx}, \frac{dP}{dx} = -5.0$ leads to an increase in the magnitude of the skin friction τ_x along both the lower and upper plates.

Rotation parameter Er An increase in the rotation parameter $Er = 0.5$ leads to an increase in the magnitude of the skin friction τ_x along both the lower and upper plates.

Time t An increase in the time t, $t = 0.45$ leads to an increase in the magnitude of the skin friction τ_x along both the lower and upper plates.

f) Skin friction τ_z normal to the plates

The skin friction at the lower plate is negative while that at the upper plate is positive except for the negative pressure gradient. We discuss the effect of changing each of the parameters on the skin friction τ_z with reference to the table 4.6 on the previous page.

Hall parameter m An increase in the value of the hall parameter $m, m = 2.0$ leads to an increase in the magnitude of the skin friction τ_z at both the lower and upper plates.

Suction parameter S Removal of suction $S, S = 0$ leads to an increase in the magnitude of the skin friction τ_z at both the lower and upper plates.

Pressure gradient A negative pressure gradient leads to a reversal in the direction of the skin friction τ_z at both the lower and upper plates.

Rotation parameter Er An increase in the rotation parameter $Er = 0.5$ leads to a decrease in the magnitude of the skin friction τ_z at both the lower and upper plates.

Time t An increase in the time t, $t = 0.45$ leads to an increase in the magnitude of the skin friction τ_z at both the lower and upper plates.

g) Rate of heat transfer Nu

The rate of heat transfer Nu is negative at the lower plate and positive at the upper plate. We discuss the effect of each of the parameters on the rate of heat transfer Nu with reference to the table 4.7 on page 117.

Hall parameter m An increase in the value of the hall parameter $m, m = 2.0$ leads to a decrease in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Suction parameter S Removal of suction $S, S = 0$ leads to a decrease in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Eckert number Ec A decrease in value of the Eckert number $Ec, Ec = 0.02$ leads to a decrease in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Pressure gradient A negative pressure gradient leads to an increase in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Rotation parameter Er An increase in the rotation parameter $Er, Er = 0.5$ leads to an increase in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

Time t An increase in the time t, $t = 0.45$ leads to an increase in the magnitude of the rate of heat transfer Nu at both the lower and upper plates.

h) Rate of mass transfer Sh

The rate of mass transfer Sh is affected by only two parameters namely the Schmidt number Sc and time t. We use table 4.8 to discuss the effect of changing each of the parameters on the rate of mass transfer.

Schmidt number Sc An increase in the Schmidt number Sc, $Sc = 0.7$ leads to a slight decrease in the magnitude of the rate of mass transfer Sh at both the lower and upper plates as observed from table 4.8 on page 117.

Time An increase in the time t, $t = 0.45$ leads to an increase in the magnitude of the rate of mass transfer Sh at both the lower and upper plates.

The conclusion made about the study that has been carried out is given in the next chapter. The recommendations to areas of further research are also given in this chapter.

Chapter 5

CONCLUSION AND RECOMMENDATIONS

In this chapter, a conclusion is given with reference to the results obtained in chapters three and four. Recommendations to further areas of research as well as some published work are also given in this chapter.

5.1 Conclusion

The study that we have carried out considered the combined effects of suction, injection and magnetic field while putting into account the Hall current effect in a MHD fluid flow between two parallel straight plates. The system is further considered to be rotating. One objective of this study was to investigate the effect of various fluid flow parameters on the velocity, temperature and concentration profiles. Another objective was to study how these parameters affect the skin friction, the rate of heat transfer and the rate of mass transfer at the plates. The parameters that were considered included the Hall parameter m, the Schimdt number Sc, the suction parameter S, the Eckert number Ec, the pressure gradient $\frac{dP}{dx}$, the rotation parameter Er and the time t. We gradually came up with the fluid flow model by beginning with a simple model of the fluid flow and then building on it. In chapter two a general fluid flow between two parallel plates with constant suction and injection was considered. The cases of a fluid flow between horizontal plates and later a fluid flow between vertical plates was analysed. Chapter three involved the study of an electrically conducting fluid between two parallel plates with one plate moving and the other plate being stationary. The effects of constant suction and injection and Hall current were also considered. Later on in chapter four, we considered a related model but this time with both plates moving in opposite directions. There were three cases considered in this chapter. The first case was that of a hydromagnetic fluid flow between two horizontal parallel plates with suction, injection and Hall current effects. Then a case of MHD fluid flow between two vertical parallel plates with the effects of suction, injection and Hall current. Finally a MHD fluid flow between two horizontal parallel plates with suction, injection and Hall current effects was considered with the whole system rotating with a constant angular velocity. In the two cases involving a fluid flow between two parallel horizontal plates, we considered the effects of the parameters mentioned above on the skin friction, the rate of heat transfer and the rate of mass transfer.

From chapter three it was observed that the primary and secondary velocity profiles near the stationary plate were affected by varying the parameters for the case of horizontal plates but not for the vertical plates. On the other hand the fluid flow near the moving plate experience the effect of varying the various parameters for both the horizontal and vertical plate systems. It was observed from chapter four that the primary and secondary velocity profiles, the temperature profiles and concentration profiles where both plates were moving were affected for the cases of horizontal plates as well as for the vertical plates. We conclude by discussing the effect of each of these parameters on the velocity, temperature and concentration profiles, the skin friction and the rates of heat and mass transfer.

Increase in the Hall parameter m

This leads to an increase in the primary velocity profiles and a decrease in the secondary velocity profiles near both plates for the moving horizontal plates with or without rotation which is similar to the case for horizontal and vertical plates where one plate was moving. There is initially an increase in the primary velocity near the plate moving with negative velocity for vertical plates but soon after the velocities start decreasing. The increase in the primary velocity profiles for vertical plates is slight near the plate moving with positive velocity. This is due to an increase in the cyclotron frequency and/or electron collision times. Further an increase in the Hall parameter leads to a decrease in the effective conductivity which reduces the magnetic dumping force on the velocity leading to an increase in the velocity.

There is a decrease in the temperature profiles near the horizontal and vertical moving plate when one plate is moving. Similarly there is a decrease in the temperature profiles near both plates for horizontal moving plates with or without rotation. The temperature also decreases near the plate moving in the negative direction for moving vertical plates. There is a decrease in the skin friction τ_x along each plate for the moving horizontal plates with or without rotation and an increase in the skin friction τ_z in the perpendicular direction for both cases when the system is rotating and when it is not rotating. The rate of heat transfer Nu decreases near all the plates for the moving horizontal plates both with and without rotation. This represents a reduction in the rate of convective heat transfer near the plates.

Removal of suction

Removal of suction leads to an increase in both the primary and secondary velocity profiles near both plates for the horizontal plates with one plate moving but a decrease in the velocity profiles for the vertical plates. There is a decrease in both the primary and secondary velocity profiles for horizontal moving plates without rotation. When the horizontal plate system is rotated, the primary and secondary velocities decrease except the primary velocities near the lower plate that increase. The primary and secondary velocity profiles decrease near the plates for vertical plates except the primary velocity profiles near the plate moving with negative velocity which increase. The temperature profiles near both plates decrease for horizontal moving plates with or without rotation. The temperature profiles for the moving vertical plates increase near the plate moving with negative velocity but decrease near the plate moving with positive velocity. The temperature profiles increase near the moving plate for the horizontal plates but decrease near the moving plate for vertical plates when one of the plates was moving. There is a decrease in the concentration profiles near the upper plates for horizontal moving plates with or without rotation and also near the plate moving with positive velocity for vertical moving plates. This is the same observation near the moving plate for vertical plates when one plate is moving but there is an increase in the concentration profiles for the horizontal plates when one plate is moving. There is decrease in the skin friction τ_x along each plate for the horizontal plates with or without rotation, an increase in the skin friction τ_z in the perpendicular direction for the case when the system is or is not rotating. The rate of heat transfer Nu decreases near all the plates for horizontal plates both with and without rotation. This represents a reduction in the rate of convective heat transfer near the plates.

Negative pressure gradient This leads to an increase in the primary and secondary velocities near both plates for the horizontal plates when one plate is moving. There is a decrease in the primary and secondary velocities near the lower plate for the moving horizontal plates and an increase in these velocities near the upper plate. There is an increase in the secondary velocity profiles and primary velocity profiles near the upper plate for horizontal moving plates in a rotating system. The primary velocity profiles near the lower plate for a rotating system are however different in that they initially decrease then they increase. The secondary velocity profiles near the lower plate decreases.

The temperature profiles decrease near the moving plate for horizontal plates when one plate is moving. The temperature profiles near both plates for horizontal plates decrease either with a rotating system or without rotation. There is an increase in the skin friction τ_x along each plate for the horizontal plates with or without rotation. The value of the skin friction τ_z in the perpendicular direction for both cases when the system is rotating and when it is not rotating reverses with a negative pressure gradient. The rate of heat transfer Nu increases near all the plates for horizontal plates both with and without rotation. This represents an increase in the rate of convective heat transfer near the plates.

Eckert number Ec A decrease in the Eckert number Ec leads to a decrease in the temperature profiles near both plates for moving horizontal plates with or without rotation. This is the same observation for the profiles near the moving plate for both horizontal and vertical plates when one plate is moving. An increase in Ec leads to an increase in the temperature profiles for moving vertical plates which is the same effect for the moving horizontal plates with or without rotation.

For the moving vertical plates, an increase in Ec leads to a decrease in the primary and secondary velocity profiles except near the plate moving with positive velocity where the velocity which was decreasing starts increasing. There is also an increase in the concentration profiles near this plate. A decrease in the Eckert number leads to an increase in the primary and secondary profiles near the moving plate for vertical plates when one plate is moving. The rate of heat transfer Nu decreases near all the plates for horizontal plates both with and without rotation for decreasing value of Ec. This represents a reduction in the rate of convective heat transfer near the plates.

Increase in the Schmidt number Sc

An increase in the Schmidt number leads to a decrease in the concentration profiles near the moving plates for both horizontal and vertical plates. For the moving horizontal plates either with or without rotation, this leads to a decrease in the concentration profiles near the upper plate. There is a decrease in the primary velocity profiles as well as the concentration profiles for the flow between vertical plates near the plate moving with positive velocity. There is decrease in the secondary velocity profiles near both plates for the vertical plates. The Schmidt number is directly proportional to the shear stresses which causes the decrease in the velocity profiles. Further the Schmidt number is inversely proportional to the concentration which explains the decrease in the concentration profiles. The rate of mass transfer Sh decreases slightly near all the plates for horizontal plates both with and without rotation. This represents a reduction in the rate of convective mass transfer near the plates possibly due to the decreased concentration.

Increase in the rotation parameter Er For a rotating system of a fluid flow between two moving horizontal plates we observe that an increase in Er leads to a decrease in the primary velocity profiles near both plates, an increase in the secondary velocity profiles near both plates and an increase in the temperature profiles near the upper plate. There is an increase in the skin friction τ_x along each plate for the horizontal plates with rotation, a decrease in the skin friction τ_z in the perpendicular direction for the case when the system is rotating and an increase in the rate of heat transfer Nu near both plates for horizontal plates with rotation. The increased skin friction represent an increase in the shear stress by the fluid on the plates. The increased rate of heat transfer represents an increase in the rate of convective heat transfer near the plates.

Increase in the time t This leads to an increase in the primary and secondary velocity profiles, the temperature profiles and the concentration profiles near the plates for all cases that are considered. This can be supported by the fact that the time is directly proportional to each of these quantities.

5.2 Recommendations

In this study we considered the flow of an electrically conducting fluid between parallel plates in the presence of a strong magnetic field whereby the system was rotating. The flow was considered to be fully developed and therefore laminar. The present work can provide a basis for further research while including the following considerations:

- Applied magnetic field inclined at an angle
- Variable magnetic field
- Fluid flow between semi finite plates
- Variable suction/ injection
- Variable fluid viscosity
- Fluid flow in the turbulent boundary layer

5.3 Research papers

- Kinyanjui M., Kirima E.M., Kwanza J.K., and Abonyo J.O. Investigations of the skin friction and the rates of heat and mass transfer for a MHD fluid flow in a rotating system. Submitted to JAGST for publication.

- Abonyo, J.O., Kinyanjui, M.N, and Mwenda, E.(2008) Investigating the effects of Hall and Ion slip currents on convective flow in rotating fluid with wall temperature oscillations. *JAGST* **10**, 103-122.
- Abonyo, J.O., Kinyanjui, M.N, and Mwenda, E. Finite difference Analysis of MHD stokes problem for a heat generating fluid with Hall current published in the proceeding of the International conference of Engineering and Mathematics, ENMA 2006 held in Bilbao, Spain, July 10-11, 2006.

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