Applied Mathematical Sciences, Vol. 12, 2018, no. 18, 879 - 891
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ams.2018.8691

Parameter Estimation of Power Lomax Distribution Based on Type-II Progressively Hybrid Censoring Scheme

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Abstract

In this study, we consider the parameter estimation of a three-parameter continuous distribution, namely, power Lomax distribution proposed by [7], when the lifetime experiments are under Type-II Progressively Hybrid censoring scheme. Expectation-Maximization algorithm was used to compute the Maximum Likelihood Estimators. Simulation was used to evaluate the performance of the maximum likelihood estimates in terms of average biases and root mean square errors.

Mathematics Subject Classification: 91G20, 91G30

Keywords: Power Lomax distribution, Maximum likelihood estimation, EM algorithm, Type-II progressively hybrid censoring scheme

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1 Introduction

The Lomax distribution proposed by [12] as a kind of Pareto-II was introduced originally for modelling business data and has been widely applied in a variety of contexts, thanks to its flexibility. In lifetime models, it is considered as an important model and belongs to the family of decreasing failure rate (see [5]). [4] found that this distribution can be used as heavy tailed alternative to the exponential, Weibull and gamma distributions. Further, it is related to the Burr family of distributions. Many authors have proposed extensions of the Lomax distribution and the three-parameter continuous distribution which we have power Lomax ($POLO$) distribution developed by [7], is one of them. $POLO$ distribution accommodates both decreasing and inverted bathtub hazard rate that is required in various survival analysis. Some works have been done using this distribution such as the recurrence relations between the single and the product moments of the K-th upper record values from $POLO$ distribution,introduced by [1]; recurrence relations between the single and the product moments of the order statistics from $POLO$ distribution proposed by [2]. In reliability and lifetime experiments, it is difficult to collect a sufficient number of observations or to observe continuously occurrence of a given event (incomplete data, which are most of time censored). On the other hand, censoring is considered in order to save time and reduce the number of failed items. Depending on the circumstances, there is a kind of censoring and the two most common censoring schemes are the type I and type II censoring schemes. The so-called Type-I censoring scheme describes the situation where the experiment continues up to a pre-specified time T. On the other hand, the Type-II censoring scheme requires the experiment to continue until a pre-specified number of failures occurs. [8] introduced the hybrid censoring scheme, which is the mixture of Type-I and Type-II censoring schemes and considered the situation where the lifetime follows the exponential distribution. One of the drawbacks of Type-I, Type-II and Hybrid censoring schemes is that they do not allow the removal of units at points other than the terminal point of the experiment. To overcome this problem, the progressive censoring schemes has been introduced few years ago which allow the experimenter to remove units before the end of the experiment.

In this work, we consider Type-II progressive hybrid censoring scheme introduced by [11] like a mixture of type-I and type-II progressive censoring schemes. This censoring allows the removal of units during the experiment and ensures that the length of the experiment can not exceed a pre-specified time point T. We suppose that n independent items are put on a life-testing experiment at the same time and the lifetimes of the n items are denoted by $X_1, ..., X_n$. The number $m < n$ of complete failures observed are fixed at the beginning of the experiment, and $R_1, ..., R_m$ are m pre-fixed integers satisfying

$R_1 + ... + R_m + m = n$. At the time of first failure $X_{1:m:n}$, R_1 of the remaining units are randomly removed. Similarly at the time of the second failure $X_{2:m:n}$, R_2 of the remaining units are removed and so on. If the $m-th$ failure $X_{m:m:n}$ occurs before the time point T, the experiment stops at the time point $X_{m:m:n}$. On the other hand suppose the $m-th$ failure does not occur before time point T and only J failures occur before the time point T , where $0 \leq J < m$, then at the time point Tall the remaining R_J^* units are removed and the experiment terminates at the time point T, where $R_J^* = n - (R_1 + ... + R_J) - J$. Hence, under Type-II progressively hybrid censoring scheme, we have one of the following types of observations;

$$Case\ I \ : \ \{X_{1:m:n}, ..., X_{m:m:n}\} \ ; \ if \ X_{m:m:n} < T, \ or \tag{1}$$
$$Case\ II \ : \ \{X_{1:m:n}, ..., X_{J:m:n}\} \ ; \ if \ X_{J:m:n} < T < X_{J+1:m:n}. \tag{2}$$

Note that for Case II, $X_{J:m:n} < T < X_{J+1:m:n} < ... < X_{m:m:n}$ and $X_{J+1:m:n}, ..., X_{m:m:n}$ are not observed.

[9] showed that under type-II progressive censoring, the Expectation - Maximization (EM) algorithm outperforms the Newton Raphson method when the data follows Lomax distribution. [10] considered also the type-I progressively hybrid censoring scheme under the assumption that the lifetime follows the Burr XII distribution. They have used the EM algorithm to compute the maximum likelihood estimates (MLEs) of the model parameters.

In this paper Maximum Likelihood estimation of the three-parameter *POLO* distribution is considered under Type II progressive hybrid censoring scheme. Expectation-Maximization (EM) algorithm is used to compute the maximum likelihood estimates (MLEs).

2 Parameter Estimation

2.1 Model Description

The random variable X is said to have *POLO* distribution, if the probability density function (PDF) is of the following form;

$$f(x) = \alpha\beta\lambda^\alpha x^{\beta-1} \left(\lambda + x^\beta\right)^{-\alpha-1}, \ x > 0, \ \alpha, \beta, \lambda > 0. \tag{3}$$

and the cumulative distribution function (CDF) is of the following form;

$$F(x) = 1 - \lambda^\alpha \left(\lambda + x^\beta\right)^{-\alpha}, \ x > 0, \ \alpha, \beta, \lambda > 0. \tag{4}$$

2.2 Maximum Likelihood Estimators

Given a type-II progressive hybrid censoring sample, the likelihood function for the two different cases are as follows

$$Case\ I \quad : \quad L(\theta) = C_1 \prod_{i=1}^{m} f(x_{i:m:n}) \left[1 - F(x_{i:m:n})\right]^{R_i}. \tag{5}$$

$$Case\ II \quad : \quad L(\theta) = C_2 \prod_{i=1}^{J} f(x_{i:m:n}) \left[1 - F(x_{i:m:n})\right]^{R_i} \left[1 - F(T)\right]^{R_J^*}. \tag{6}$$

where, $C_1 = \prod_{i=1}^{m} \left[n - \sum_{k=1}^{i-1}(1 + R_k)\right]$, $C_2 = \prod_{i=1}^{J} \left[n - \sum_{k=1}^{i-1}(1 + R_k)\right]$ and $R_J^* = n - (R_1 + ... + R_J) - J$.

Let $\theta = (\alpha, \beta, \lambda)$ and according to the two cases defined above,ignoring the constants, the joint likelihood function is given as follows;

Case I:

$$L_1(\theta) = (\alpha\beta)^m \lambda^{n\alpha} \left(\prod_{i=1}^{m} x_i^{\beta-1}\right) \left(\prod_{i=1}^{m} \left(\lambda + x_i^{\beta}\right)^{-\alpha(1+R_i)-1}\right). \tag{7}$$

Case II:

$$L_2(\theta) = (\alpha\beta)^J \lambda^{n\alpha} \left(\prod_{i=1}^{J} x_i\right)^{\beta-1} \left(\prod_{i=1}^{J} \left(\lambda + x_i^{\beta}\right)^{-\alpha(R_i+1)-1}\right) \left(T^{\beta} + \lambda\right)^{-\alpha R_J^*}. \tag{8}$$

Combining (7) and (8),we get the following likelihood function;

$$L(\theta) \propto (\alpha\beta)^D \lambda^{n\alpha} \left(\prod_{i=1}^{D} x_i\right)^{\beta-1} \left(\prod_{i=1}^{D} \left(\lambda + x_i^{\beta}\right)^{-\alpha(R_i+1)-1}\right) \left(T^{\beta} + \lambda\right)^{-\alpha R_D^*}.$$

Where, for case I, $D = m$, $R_m^* = 0$ and for case II, $D = J$, $R_J^* = n - \sum_{i=1}^{J-1} R_i - J$.

Hence, the combined log-likelihood is given by:

$$l(\theta) = Dln\alpha + Dln\beta + n\alpha ln\lambda + (\beta-1)\sum_{i=1}^{D} ln(x_i) - \sum_{i=1}^{D}[\alpha(R_i+1)+1]ln(\lambda + x_i^{\beta}) - \alpha R_D^* ln(\lambda + T^{\beta}). \tag{9}$$

Differentiating equation (9) with respect to $\alpha, \beta\ and\ \lambda$, we get the following normal equations

$$\begin{cases} \frac{D}{\alpha} + nlog\lambda - \sum_{i=1}^{D}(R_i+1)log(\lambda + x_i^{\beta}) - R_D^{*}log(\lambda + T^{\beta}) = 0 \\ \frac{D}{\beta} + \sum_{i=1}^{D} logx_i - \sum_{i=1}^{D} \frac{x_i^{\beta}logx_i[\alpha(R_i+1)+1]}{\lambda + x_i^{\beta}} - \frac{\alpha R_D^{*} T^{\beta} logT}{\lambda + T^{\beta}} = 0 \\ \frac{n\alpha}{\lambda} - \sum_{i=1}^{D} \frac{\alpha(R_i+1)+1}{\lambda + x_i^{\beta}} - \frac{\alpha R_D^{*}}{\lambda + T^{\beta}} = 0 \end{cases} \quad (10)$$

It is obvious that the solution of the system (10) are not in the closed forms.

2.3 EM Algorithm

The EM algorithm proposed by [6] is an iterative procedure for computing the maximum likelihood estimators when we are dealing with incomplete data. Estimating the unknown parameters of *POLO* distribution under Type-II progressively hybrid censoring can be viewed as an incomplete data problem. We can only observe the complete failure times of D units as $X_{1:m:n}, X_{2:m:n}, ..., X_{D:m:n}$ in the Type-II progressively hybrid censoring experiment. We denote by $X = \{X_{1:m:n}, X_{2:m:n}, ..., X_{D:m:n}\}$ and $Z = \{Z_{ij},\ j = 1, 2, ..., R_i; i = 1, 2, ..., D\} \cup \{Z_{Tj}, j = 1, 2, ..., R_D^{*}\}$ the observed and missing data, respectively. Where $Z_{ij},\ j = 1, 2, ..., R_i; i = 1, 2, ..., D$ stands for the j-th censored variables at the failure time $X_{i:m:n}$, and $Z_{Tj}, j = 1, 2, ..., R_D^{*}$ denotes the j-th censored variables at the failure time T. So, the complete data can be denoted as $W = (X, Z)$ and the joint density function of complete data based on Type-II progressively hybrid censoring scheme is given by;

$$L_c(\theta) = \prod_{i=1}^{D}\left[f(x_{i:m:n})\prod_{j=1}^{R_i} f(z_{ij})\right]\prod_{j=1}^{R_D^{*}} f(z_{Tj}). \quad (11)$$

Using (11), the complete log likelihood function of POLO distribution based on type-II progressive hybrid censoring is given as

$$\begin{aligned} l_c(\theta) \quad = \quad & nlog\alpha + nlog\beta + n\alpha log\lambda + (\beta - 1)\sum_{i=1}^{D} log(x_i) - (\alpha + 1)\sum_{i=1}^{D} log(\lambda + x_i^{\beta}) \\ + \quad & (\beta + 1)\sum_{i=1}^{D}\sum_{j=1}^{R_i} log(z_{ij}) - (\alpha + 1)\sum_{i=1}^{D}\sum_{j=1}^{R_i} log(\lambda + z_{ij}^{\beta}) + (\beta + 1)\sum_{j=1}^{R_D^{*}} log(z_{Tj}) \\ - \quad & (\alpha + 1)\sum_{j=1}^{R_D^{*}} log(\lambda + z_{Tj}^{\beta}). \end{aligned}$$

The E-step involves the computation of the conditional expectation $E_{\theta^{(k)}}\left[l_c(\theta) \mid X = x, \theta^{(k)}\right]$ which we call pseudo log-likelihood function $Q(\theta;\theta^{(k)})$ defined as

$$\begin{aligned} Q(\theta|\theta^{(k)}) &= nlog\alpha + nlog\beta + n\alpha log\lambda + (\beta-1)\sum_{i=1}^{D} log(x_i) - (\alpha+1)\sum_{i=1}^{D} log(\lambda + x_i^{\beta}) \\ &+ (\beta+1)\sum_{i=1}^{D}\sum_{j=1}^{R_i} E\left[log(z_{ij}) \mid z_{ij} > x_i\right] - (\alpha+1)\sum_{i=1}^{D}\sum_{j=1}^{R_i} E\left[log(\lambda + z_{ij}^{\beta}) \mid z_{ij} > x_i\right] \\ &+ (\beta+1)\sum_{j=1}^{R_D^*} E\left[log(z_{Tj}) \mid z_{Tj} > T\right] - (\alpha+1)\sum_{j=1}^{R_D^*} E\left[log(\lambda + z_{Tj}^{\beta}) \mid z_{Tj} > T\right]. \end{aligned}$$

In Type-II progressive hybrid censoring, we denote the conditional PDF of all censored data z_{ij}, for $i = 1, 2, ..., D$, $j = 1, 2, ..., R_i$ and z_{Tj}, for $j = 1, 2, ..., R_D^*$, respectively, as follows:

$$f_{Z/X}(z_{ij}) = \frac{f(z_{ij})}{1 - F(X_{i:m:n})}, \; z_{ij} > x_{i:m:n}. \tag{12}$$

and

$$f_{Z/X}(z_{Tj}) = \frac{f(z_{Tj})}{1 - F(T)}, \; z_{Tj} > T. \tag{13}$$

Using (12) and (13), we compute the expectations which are in the pseudo log-likelihood function. It follows that,

$$\begin{aligned} E\left[log(z) \mid z > x_i\right] &= \frac{\alpha\beta\lambda^{\alpha}}{1 - F(x_i)}\left[\frac{\left(\lambda + x_i^{\beta}\right)^{-\alpha} log(x_i)}{\alpha\beta} + K_1\right]. \\ &= A_1(x_i;\theta). \end{aligned}$$

where

$$K_1 = \frac{1}{\alpha\beta^2\lambda^{\alpha}}\int_0^{c_i} y^{\alpha-1}(1-y)^{-1}dy \qquad with \;\; c_i = \left(1 + \frac{x_i^{\beta}}{\lambda}\right)^{-1}.$$

By analogy,

$$\begin{aligned} E\left[log(z_{Tj}) \mid z_{Tj} > T\right] &= \frac{\alpha\beta\lambda^{\alpha}}{1 - F(T)}\left[\frac{\left(\lambda + T^{\beta}\right)^{-\alpha} log(T)}{\alpha\beta} + K_2\right]. \\ &= A_2(T,\theta). \end{aligned}$$

Where,

$$K_2 = \frac{1}{\alpha\beta^2\lambda^{\alpha}}\int_0^{c} y^{\alpha-1}(1-y)^{-1}dy \qquad with \;\; c = \left(1 + \frac{T^{\beta}}{\lambda}\right)^{-1}.$$

On the other hand, we have

$$\begin{aligned} E\left[log(\lambda + z_{ij}^{\beta}) \mid z_{ij} > x_{i:m:n}\right] &= \frac{1}{\alpha} + log(\lambda + x_i{}^{\beta}). \\ &= B_1(x_i;\theta). \end{aligned}$$

By analogy,

$$\begin{aligned} E\left[log(\lambda + z_{Tj}^{\beta}) \mid z_{ij} > T\right] &= \frac{1}{\alpha} + log(\lambda + T^{\beta}). \\ &= B_2(T;\theta). \end{aligned}$$

Therefore, the pseudo log-likelihood function becomes

$$\begin{aligned} Q(\theta|\theta^{(k)}) \quad = \quad & nlog\alpha + nlog\beta + n\alpha log\lambda + (\beta-1)\sum_{i=1}^{D} log(x_i) - (\alpha+1)\sum_{i=1}^{D} log(\lambda + x_i^{\beta}) \\ + \quad & (\beta-1)\sum_{i=1}^{D} R_i A_1(x_i;\theta^{(k)}) - (\alpha+1)\sum_{i=1}^{D} R_i B_1(x_i;\theta^{(k)}) + (\beta-1)R_D^* A_2(T;\theta^{(k)}) \\ - \quad & (\alpha+1)R_D^* B_2(T;\theta^{(k)}). \end{aligned}$$

The M-step involves the calculation of the estimate $\theta^{(k+1)} = (\alpha^{(k+1)}, \beta^{(k+1)}, \lambda^{(k+1)})$ of $\theta = (\alpha, \beta, \lambda)$ by maximizing $Q(\theta|\theta^{(k)})$, where $\theta^{(k)}$ is an estimate of θ at the k^{th} iteration.
Then, $\theta^{(k+1)} = (\alpha^{(k+1)}, \beta^{(k+1)}, \lambda^{(k+1)})$ is given by the following system

$$\begin{cases} \frac{n}{\alpha} + nlog\lambda - \sum_{i=1}^{D} log(\lambda + x_i^{\beta}) - \sum_{i=1}^{D} R_i B_1(x_i;\theta^{(k)}) - R_D^* B_2(T;\theta^{(k)}) = 0 \\ \frac{n}{\beta} + \sum_{i=1}^{D} log(x_i) - (\alpha+1)\sum_{i=1}^{D} \frac{x_i^{\beta} log(x_i)}{\lambda + x_i^{\beta}} + \sum_{i=1}^{D} R_i A_1(x_i;\theta^{(k)}) + R_D^* A_2(T;\theta^{(k)}) = 0 \\ \frac{n\alpha}{\lambda} - (\alpha+1)\sum_{i=1}^{D} \frac{1}{\lambda + x_i^{\beta}} = 0 \end{cases}$$

Given $\beta^{(k+1)}$, $\lambda^{(k+1)}$ and using the first equation in the above system, $\alpha^{(k+1)}$ is given such as

$$\alpha^{(k+1)} = \frac{n}{\sum_{i=1}^{D} log(\lambda^{(k+1)} + x_i^{\beta^{(k+1)}}) + \sum_{i=1}^{D} R_i B_1(x_i;\theta^{(k)}) + R_D^* B_2(T;\theta^{(k)}) - nlog\lambda^{(k+1)}}. \tag{14}$$

We can see clearly, $\beta^{(k+1)}$ and $\lambda^{(k+1)}$ are not in the closed form. In that case, [6] proposed the generalized EM algorithm (GEM algorithm) for which the M-step requires $\theta^{(k+1)}$ to be chosen such that

$$Q(\theta^{(k+1)}|\theta^{(k)}) \geq Q(\theta^{(k)}|\theta^{(k)}),\ where\ \theta = (\alpha, \beta, \lambda). \tag{15}$$

In this work, we are using the EM gradient algorithm which approximates the M-step of the EM algorithm by using one step of the Newton-Raphson method.

Let us put $\alpha = h(\beta, \lambda)$ where α is define like in the equation (14). Replacing α in the (14) by $h(\beta, \lambda)$, we obtained the profile pseudo log-likelihood function $Q^*(\theta^*|\theta)$ where $\theta^* = (\beta, \lambda)$ and $\theta = (\alpha, \beta, \lambda)$. It is given by

$$\begin{aligned} Q^*(\theta^*|\theta^{(k)}) &= nlog\,(h(\beta, \lambda)) + nlog\beta + nh(\beta, \lambda)log\lambda + (\beta-1)\sum_{i=1}^{D} log(x_i) \\ &- (h(\beta, \lambda)+1)\sum_{i=1}^{D} log(\lambda + x_i^{\beta}) + (\beta-1)\sum_{i=1}^{D} R_i A_1(x_i;\theta^{(k)}) \\ &- (h(\beta, \lambda)+1)\sum_{i=1}^{D} R_i B_1(x_i;\theta^{(k)}) + (\beta-1)R_D^* A_2(T;\theta^{(k)}) \\ &- (h(\beta, \lambda)+1)R_D^* B_2(T;\theta^{(k)}). \end{aligned}$$

So that, in the M-step, instead of to apply the one step of the Newton-Raphson on $Q(\theta|\theta^{(k)})$, we apply it on $Q^*(\theta^*|\theta^{(k)})$ and we compute the estimate of α at the $(k+1)^{th}$ iteration as follows:

$$\widehat{\alpha}^{(k+1)} = h(\widehat{\beta}^{(k+1)}, \widehat{\lambda}^{(k+1)}).$$

3 Simulation study

In this section, simulations are conducted in order to illustrate the performance of the EM algorithm for different combinations of sample sizes and censoring schemes. Consider the following three censoring schemes:
Scheme 1: $R_1 = R_2 = \cdots = R_{m-1} = 0$, $R_m = n - m$,
Scheme 2: $R_1 = n - m$, $R_2 = R_2 = \cdots = R_{m-1} = 0$,
Scheme 3: $R_1 = R_2 = \cdots = R_{m-1} = 1$, $R_m = n - 2m + 1$
under three different time-points: $T_1 = X_{[\frac{m}{3}]:m:n} + 0.01$, $T_2 = X_{[\frac{2m}{3}]:m:n} + 0.01$ and $T_3 = X_{m:m:n} + 1$, where $[\pi]$ denotes the integer part of the positive number π. The algorithm in [3] is used to generate a Type-II progressive hybrid censored sample from $POLO$ distribution for the parameter value $\alpha = 1$, $\beta = 2$, and $\lambda = 1$. This algorithm is described as follows:

Step 1: Generate m independent and identically distributed random numbers $(U_1, ..., U_m)$ from uniform distribution U[0,1];
Step 2: For $i = 1, 2, ..., m$, set $Z_i = -log(1 - U_i)$, then $Z_i's$ are independent and identically distributed standard exponential distribution variables;
Step 3: Given n, m and the censoring scheme $R = (R_1, ..., R_m)$, obtain a Type-II progressive censored sample $(E_1, E_2, ..., E_m)$ from standard exponential distribution using the following system

$$\begin{aligned} E_1 &= \tfrac{Z_1}{n}, \\ E_i &= E_{i-1} + \tfrac{Z_i}{n - \sum_{j=1}^{i-1} R_j - i + 1}, \ i = 2, 3, ..., m. \end{aligned}$$

Step 4: For $i = 1, 2, ..., m$, set $Y_i = 1 - exp(-E_i)$, then $Y_i's$ form a Type-II progressive censored data from uniform distribution U[0,1];
Step 5: For $i = 1, 2, ..., m$, set $X_{i:m:n} = F^{-1}(Y_i)$, where $F^{-1}(\cdot)$ is the inverse CDF of *POLO* distribution. So that $X_i's$ form a Type-II progressive censored data from *POLO* distribution.
If $X_{m:m:n} \leq T$ which is case I, then $(X_{1:m:n}, R_1), (X_{2:m:n}, R_2), ..., (X_{m:m:n}, R_m)$ is called Type-II progressive hybrid censored data of *POLO* distribution.
Otherwise, if $X_{m:m:n} > T$ which is case II, then $(X_{1:m:n}, R_1), (X_{2:m:n}, R_2), ..., (X_{J:m:n}, R_J)$, defined the Type-II progressive hybrid censored data, where J is find such that $X_{j:m:n} < T < X_{j+1}$.

The simulation process of estimation is executed M times ($M = 1000$) and the criteria used for evaluation are average bias and root means squared error (RMSE) of estimates, which are computed as:

$$\begin{aligned} Bias &= \frac{1}{M}\sum_{i=1}^{M}(\widehat{\theta}_{EM_i} - \theta). \\ RMSE &= \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(\widehat{\theta}_{EM_i} - \theta\right)^2}. \end{aligned}$$

Where, we suppose $\widehat{\theta}_{EM_i}$ is the estimate of θ for the i^{th} simulated data set. Note that the initial guess values are considered to be the same as the true parameter values.

The results of the simulation study are presented in Tables 1-3. From these tables, we can see that, globally, the EM algorithm is quite efficient for *POLO* distribution under Type-II progressive hybrid censored data and the proposed method underestimates the parameters. For all given sampling schemes, we observe that:

1. For fixed n and m as pre-specified time point of experiment increases, the biases and RMSEs of the estimates for most of estimated parameters decrease as expected.

2. For fixed n and T as m increases, the biases and RMSEs are decreasing.

3. For fixed m and T as n increases, the biases and RMSEs are increasing for most of the estimates.

4. For fixed n, m in Table 1, we can observe that the biases and RMSEs for most of the parameters are smaller in scheme 1, but also with scheme 2 being smaller that scheme 3 under different combinations of sample size.

Table 1: Average bias of the different estimates and the corresponding RMSE, when $(\alpha, \beta, \lambda) = (1, 2, 1)$ at time point $T_1 = X_{[m/3]} + 0.01$

		Bias			RMSE		
(n,m)	schemes	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\lambda}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\lambda}$
(30 , 10)	1	-0.349	-0.196	-0.537	0.362	0.201	0.552
	2	-0.345	-0.195	-0.542	0.363	0.200	0.564
	3	-0.338	-0.198	-0.531	0.362	0.203	0.553
(30 , 15)	1	-0.251	-0.118	-0.495	0.260	0.131	0.513
	2	-0.237	-0.099	-0.521	0.258	0.111	0.536
	3	-0.237	-0.113	-0.510	0.260	0.126	0.517
(40 , 15)	1	-0.449	-0.221	-0.706	0.481	0.239	0.721
	2	-0.473	-0.207	-0.740	0.493	0.213	0.749
	3	-0.460	-0.228	-0.713	0.482	0.237	0.723
(40 , 20)	1	-0.422	-0.168	-0.686	0.445	0.181	0.706
	2	-0.434	-0.145	-0.711	0.456	0.155	0.732
	3	-0.431	-0.164	-0.685	0.446	0.176	0.710
(70 , 30)	1	-0.455	-0.193	-0.717	0.462	0.199	0.723
	2	-0.459	-0.161	-0.746	0.466	0.166	0.750
	3	-0.460	-0.188	-0.718	0.463	0.194	0.727

5. For fixed n, m in Table 2 and Table 3, biases and RMSEs for most of the parameters in scheme 2 and scheme 3 are smaller than in scheme 1.

4 Conlusion

In this article, we discussed the estimation for parameters of power Lomax distribution when the data is coming from Type-II progressive hybrid censoring scheme. The MLEs are computed using EM algorithm. The results of the simulation study showed that the performance of EM algorithm is quite sufficient and the method underestimates the unknown parameters.

Acknowledgements. The authors would like to express their sincere gratitude to African Union for supporting this research at the Pan African University, Institute for Basic Sciences, Technology and Innovation.

Table 2: Average bias of the different estimates and the corresponding RMSE, when $(\alpha, \beta, \lambda) = (1, 2, 1)$ at time point $T_2 = X_{[2m/3]} + 0.01$

		Bias			RMSE		
(n,m)	schemes	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\lambda}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\lambda}$
(30 , 10)	1	-0.215	-0.168	-0.441	0.229	0.185	0.457
	2	-0.210	-0.111	-0.519	0.215	0.124	0.527
	3	-0.219	-0.158	-0.458	0.225	0.175	0.466
(30 , 15)	1	-0.014	-0.104	-0.338	0.065	0.139	0.346
	2	0.047	-0.004	-0.461	0.055	0.071	0.470
	3	0.021	-0.060	-0.387	0.053	0.105	0.391
(40 , 15)	1	-0.275	-0.201	-0.541	0.279	0.227	0.544
	2	-0.309	-0.076	-0.669	0.317	0.105	0.683
	3	-0.273	-0.170	-0.570	0.280	0.194	0.573
(40 , 20)	1	-0.134	-0.125	-0.467	0.140	0.165	0.474
	2	-0.168	0.004	-0.621	0.171	0.084	0.630
	3	-0.132	-0.071	-0.524	0.137	0.121	0.531
(70 , 30)	1	-0.572	-0.304	-0.866	0.580	0.344	0.870
	2	-0.167	-0.033	-0.619	0.168	0.070	0.625
	3	-0.538	-0.213	-0.843	0.586	0.259	0.860

Table 3: Average bias of the different estimates and the corresponding RMSE, when $(\alpha, \beta, \lambda) = (1, 2, 1)$ at time point $T_3 = X_{[m]} + 1$

		Bias			RMSE		
(n,m)	schemes	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\lambda}$	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{\lambda}$
(30 , 10)	1	-0.179	-0.271	-0.239	0.202	0.290	0.253
	2	-0.016	-0.048	-0.507	0.039	0.081	0.519
	3	-0.102	-0.199	-0.310	0.128	0.218	0.313
(30 , 15)	1	-0.041	-0.208	-0.109	0.155	0.232	0.115
	2	0.329	0.039	-0.456	0.336	0.079	0.467
	3	0.291	-0.038	-0.306	0.315	0.092	0.308
(40 , 15)	1	-0.194	-0.342	-0.277	0.209	0.366	0.282
	2	-0.184	-0.006	-0.678	0.185	0.078	0.684
	3	-0.114	-0.209	-0.390	0.127	0.237	0.393
(40 , 20)	1	-0.076	-0.283	-0.138	0.138	0.309	0.144
	2	0.048	0.073	-0.620	0.055	0.109	0.629
	3	0.160	-0.039	-0.409	0.174	0.107	0.414
(70 , 30)	1	-0.255	-0.586	-0.353	0.259	0.616	0.358
	2	0.136	0.024	-0.577	0.145	0.067	0.586
	3	-0.281	-0.217	-0.664	0.292	0.264	0.670

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Received: July 9, 2018; Published: July 30, 2018