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CLASSICAL CONSTRUCTION OF ANGLES IN GENERAL

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ABSTRACT

The construction of angles in general is a classical problem in mathematics. The early mathematicians failed to redress the problem under the stated restrictions because they did not have a bearing to approach the problem from. Eventually, the classical problem was assumed impossible. This paper contributes the interest in solving this crucial problem by presenting a very straight forward methodology of constructing angles in general. Several geometrical constructions were carried out to answer the questions; what methodology would generalize the construction of all angles measurable using the protractor? What approach would lead to the construction of some regular polygons whose the angle subtended at their center would only be estimated? The methods revealed in this work responded to these two questions in a simple but a more fashionable way. Two smart chords were generated which helped construct any angle a multiple of both five or two, and both five and two. The methodology involved relating the angles at a difference of ten degrees from each other in their descending order. The idea yielded excellent results. Linear simultaneous equations were used to confirm the accuracy of the two developed chords. The chords were therefore considered to form the base for constructing angles in general.

Key Words: *Classical construction, Pair of compass, Ruler, Trisection of angles, Chord, Elegant solution, Nonagon, Pentadecagon.*

1. INTRODUCTION

The construction of angles in general is a classical problem of geometry in mathematics. It concerns the partitioning of an arbitrary angle into a desired fraction, or the construction of an angle of a certain ratio. It involves one of the three problems posed by the ancient Greek mathematicians; 'trisection of angles'. Mathematicians and other practitioners have had the inspiration to be able to solve the trisection of angles but no elegant solution has been achieved by this day. However, high reputable contemporaries have closed the door in solving this problem by assuming it impossible, and so is the construction of angles in general- under the stated restrictions [[1]-[5]], [9], and [10]. The proof upon the impossibility involved the use of Galois Theory of algebra which stated; 'The trisection of an angle corresponds to the solution of a certain cubic equation which cannot be solved using a ruler (straightedge) and compass construction [1], [11]'. The methods presented to solve the problem of constructing angles in general or the cutting of an angle into a certain ratio does not follow the rules of Greek's geometry [1], [4], and [9]. The work revealed in this paper is consistent with the framework of the Euclidean construction. The deeper need upon the objectives of this innovation was boosted by the questions; what type of a triangle should subtend an angle of either 10^0 or 8^0 at a point when geometrically constructed? Would the procedure for drawing such triangle(s) generalize for the construction of all the other angles multiples of 2, 5, and both 2 and 5? The methodology used to answer the two questions yielded credible results. Two particular chords were generated after several geometric attempts, to inspect the property of some chords lying on any circular plane; the two chords would always subtend an angle of 10^0 and 8^0 respectively. The accuracy of the two chords was confirmed by the construction of some regular polygons; 9sides of $n=40^0$ and 15 sides of $n=24^0$ respectively, and n is the angle subtended at the center of the polygons. The methodology involved taking the ratio between any two angles at difference of 10^0 from each other in their descending order. The most considerable ratio was 60^0: 50^0 = 1.2:1. This was due to the fact that the 60^0 angle is the base for the construction of all the angles multiples of 15. It was assumed that, there are some uniform chords (fractions of the diameter of a circle) which redistribute along any circular plane to produce the curvature of the circle. These lengths could define the ratio between any two angles measurable using the protractor, and thus the desire to construct them.

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2. MATERIALS AND METHODS

2.1 Materials; A pair of compass, a ruler (straightedge), piece of drawing paper, pencil.

2.2 Methods

2.2.1 Method 1

The construction of a 50^0 angle using a straightedge {ruler} and a pair of compass only;

1. Draw a straight line and mark two points O and P on the line.
2. Mark a point Q equidistance OP from both O and P and draw an arc PQ centered at O.
3. Join the chord QP, and draw its bisector through a point R to cut curve PQ at S.
4. Draw the bisector of angle SRP to cut the curve PS at T.
5. Place your compass at point Q and make a small arc of length RT to cut curve SQ at U.
6. Join the point U to O and there you have angle UOP= 50^0

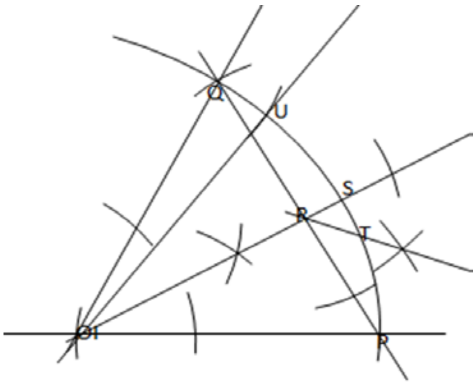

Figure-1: Constructionof 50^0 angle

2.2.2 Method 2

Construction of 8^0angle using a straightedge {ruler} and a pair of compass only.

1. Draw a straight line and mark two points O and P on the line.
2. Mark a point Q equidistance OP from both O and P and draw an arc PQ centered at O.
3. Join the chord QP, and draw its bisector through a point R to cut curve PQ at S.
4. Draw the bisector of angle SRP to cut curve PS at T.
5. Further, draw the bisector of angle SRT to cut curve ST at U.
6. Position the pair compass at point P and make an arc of length RU to cut curve PTUS at V. Angle VOP = 8^0.

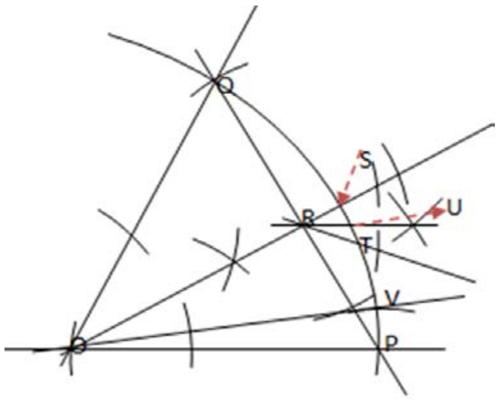

Figure-2: Construction of 8^0angle

2.2.3 Justification of the methods

2.2.3.1 Method

Construction of a nonagon to justify the construction of 10^0 angle

1. Draw a straight line and mark two points O and P on the line.
2. Draw a circle of radius OP centered at O.
3. Make a small arc of length OP to cut the circular curve at Q. Join Q to P.

4. Construct the bisector of angle QOP and extent it to cut chord QP at R and the curve PQ at S.
5. Draw the bisector of angle SRP to cut the curve PSQ at T.
6. Place your pair of compass at S, and make a small arc of length RT to cut curve SQ at U.
7. Mark equal arcs using chords of length UP along the circumference.

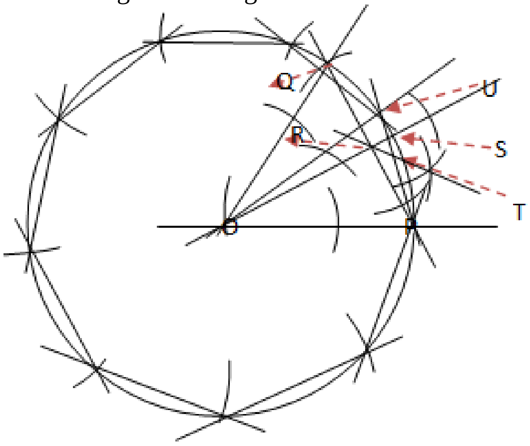

Figure-3: Construction of a nonagon

Angle SOP=30^0. Let angle UOS be M, and the base angles of the isosceles triangle UOP be Y each. It follows that, $[30+M]^0 + [2Y]^0 = 180^0$, or 2Y+M=150……………….. [1]
Using the expression for calculating the size of an interior angle of a regular polygon we have $([2n-4]90^0) / n = 2Y$ (interior angle)……… [2], for n= 9. Equation (2) gives Y=140^0 and Equation [1] gives M= 10^0 proving that angle UOP= 40^0 as required.

2.2.3.2 Method

Construction of a pentadecagon to justify the construction of 8^0 angle.

1. Draw a straight line and mark two points O and P on the line.
2. Draw a circle of radius OP centered at O.
3. Make a small arc of length OP to cut the circular curve at Q. Join Q to P.
4. Construct a bisector of angle QOP and extent it to cut chord QP at R and the curve PQ at S.
5. Draw the bisector of angle SRP to cut the curve at T.
6. Further, construct the bisector of angle SRT to cut curve TS at U.
7. Place the pair of compass at S and make a small arc of length RT to cut curve SQ at a point V.
8. Using RU, place your pair of compass at V and make another arc to cut VQ at W.
9. Join W to O and construct its bisector to cut curve PTUVS at a point Y. Mark equal arcs using chords of length YP along the circumference.

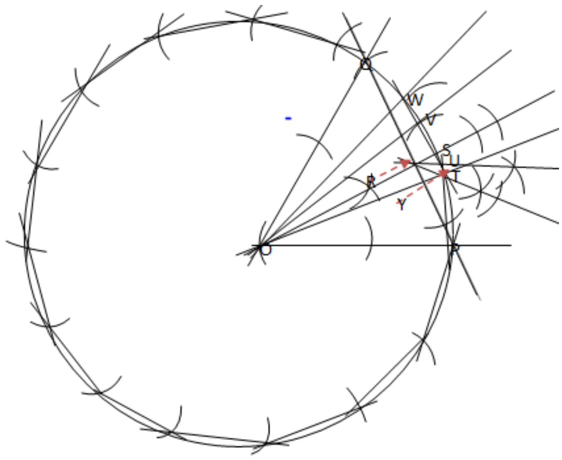

Figure-4: Construction of a pentadecagon

Angle SOP=30^0. Let angle SOY=X, then angle YOP= $[30 - X]^0$
Let the base angles of the isosceles triangle YOP be Y each. Therefore, $[30 - X]^0 + [2Y]^0 = 180^0$.
$[2Y]^0 - X^0 = 150^0$……………. [1] Using the expression; $[2n-4]90^0 \div n$, and n= 15 we get;
$[2n-4]90^0 \div n = 2Y^0$ and 2Y is size of interior angle. Y = 78^0…………….. [2], and equation [1] gives X = 6^0.
YOP= $[30-6]^0 = 24^0$, the required angle. Angle WOP is twice angle YOP implying chord RU=VW; would always subtend an angle of 8^0 at O, since SV=RT, which subtends 10 degrees at the center O.

3. RESULTS AND DISCUSSION

These results were found after a handful geometric constructions performed to inspect the characteristics of some lengths present in any circular plane in relation to the angle they would subtend at the center of a circular plane. The two methods were justified by the construction of two regular polygons; a nonagon and a pentadecagon, which subtends angles of 40^0 and 24^0 at their center respectively. The two derived chords RT and RU were assumed to be fractions of the diameter which redistributes itself along the circular plane and at some inclinations to produce the circumference. The chords RT and RU would help construct angles of 10^0 and $8\ ^0$ respectively. Thus the methods would generalize for the construction of any required angle, measurable using the protractor. The methodology involved taking the ratios of angles at a difference of 10^0 from each other in the order; 180^0:170^0, 170^0:160^0, 160^0:150^0… The most significant ratio was 60^0:50^0, since the 60^0 angle forms the base for the construction of all the angles multiples of 15 as stated earlier. If these ratios were converted into radians, they would represent the ratio of the chords PQ to PU which is 1.2: 1. Utilizing the two chords helped to solve the partitioning of an arbitrary angle to required fraction, and the vice versa was true. For instance, the lengths RT and RU could be utilized in the construction of a 1^0 angle. Figures (5) and (6) illustrates how a 1^0 and 60^0 angles could be constructed and trisected respectively;

3.1 *Construction of 1^0angle using a straightedge {ruler} and compass only.*

1. Draw a straight line and mark two points O and P on the line.
2. Mark a point Q equidistance OP from both O and P and draw an arc PQ centered at O.
3. Join the chord QP, and draw its bisector through a point R to cut curve PQ at S.
4. Draw the bisector of angle SRP to cut curve PS at T.
5. Further, draw the bisector of angle SRT to cut curve ST at U.
6. Place your pair of compass at P and make an arc of length RT to cut curve PT at V.
7. Make an arc VW of length RU with your pair of compass placed at pint V.
8. Construct the bisector of angle WOP to cut curve VW at Y. Angle YOP = 1^0.

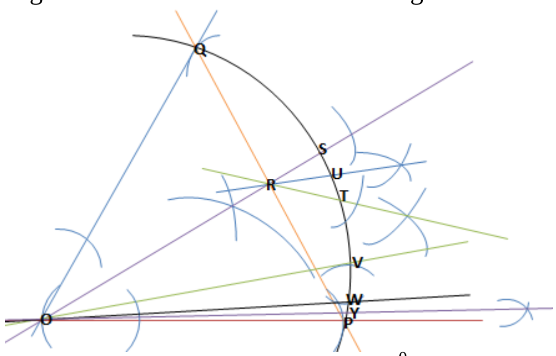

Figure-5: Construction of a 1^0 angle.

3.2 *The trisection of a 60^0 angle using a ruler and a pair of compass only.*

1. Draw a straight line and mark two points O and P on the line.
2. Mark a point Q equidistance OP from both O and P and draw an arc PQ centered at O.
3. Join the chord QP, and draw its bisector through a point R to cut curve PQ at S.
4. Draw the bisector of angle SRP to cut the curve PS at T.
5. Placing your pair of compass at S, make two arcs of length RT to cut the curve PSQ at either sides of S.
6. Joint the two arcs to O and the angle QOP would be partitioned into three equal parts.

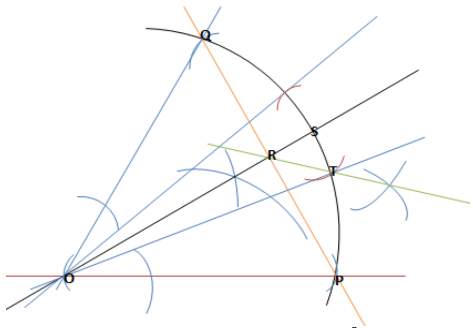

Figure-6: Trisection of a 60^0 angle.

4. CONCLUSION

The problem of constructing angles in general or their desired ratios has its roots in the ancient Greece. Through the ages, mathematicians and other professions have expended vast amounts of energy in remedying the problem (compass-ruler construction); but no precise solution has been presented. It is a problem governed by use of only two tools; a pair of compass and a ruler. However, to this date the problem is considered to be impossible to solve under the classical Greek's rules of geometry. The approaches taken to redress the problem involved manipulation and violation of the imposed limitations by the Greek mathematicians [2], [4], [9], and [10]. Moreover, the revealed methods were just approximations and none of them is absolutely accurate [9]. This work was motivated by too much curiosity by mathematicians, enthusiastic to solve the problem, and the wide applications of geometry in many fields. The main objective of this work was to frame a procedure(s) that would generalize for the construction of angles multiples of 5 and 2, and it would have the problem solved. Two chords were generated after various constructions carried out to inspect the properties of some lengths lying on any circular plane, in relation to the angles they would subtend at a point. The methodology concerned relating angles at a difference of 10^0 degrees from each other in their descending order. This idea gave considerable results. Methods (2.2.1) and (2.2.2) of this paper would help construct angles of 10^0 and 8^0 respectively. The two methods could therefore be utilized in constructing any other angle measurable using the protractor. The procedures are straightforward and precise of calculations. The accuracy of the two methods was confirmed by the construction of a regular; nonagon and pentadecagon, under the set limits by the early Greek mathematicians.

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